

IMPLICATIONS OF AN AGING POPULATION ON PENSION SYSTEMS AND FINANCIAL MARKETS

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ABSTRACT

In this paper, we introduce a macroeconomic model of overlapping generations to analyze the impacts of the demographic changes as well as the interactions between pension system, bond and stock markets. Furthermore, we examine how the pension system influences the distribution of wealth, consumption and saving within generations.

In the context of this model, we show a drastic decline of capital market returns due to an aging population. Moreover, we examine the impacts demographic changes can have on individuals' welfare for an existing pay-as-you-go pension scheme. Raising the pensionable age combined with a decrease of the contributions seems to be the best policy. On the other hand, increases in contributions as a result of demographic changes show the highest welfare losses. Taken into account the recent pension reforms in Germany, raising the retirement age or a faster transition from a pay-as-you-go pension system to a capital funded one would make sense. But it is questionable whether such a policy will be enforceable with an aging electorate.

Keywords: Pension Scheme, Demographic Change, Interest Rate, Overlapping Generations.

1. INTRODUCTION

How can pension schemes be sustainably financed? This is an important issue particularly with regard to the demographic changes in many industrialized countries but also in other emerging economies such as China and India. While the immediate concern is the financial market and currency crisis, the much more important challenge for capital markets over the medium and long-term is dealing with the implications of an

aging population on pension systems and the role that financial markets can play in providing adequate private pensions.

Demographic changes are the result of steadily increasing life expectancy due to medical improvements and falling fertility rates³. The hump-shaped profile of fertility rates has created the “baby-boom” generation; that is, a generation or cohorts which are larger than the preceding and subsequent cohorts. Therefore, the effects of the baby-boom generation

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³National fertility levels are considered „too low“ by majority of governments in developed countries, e.g. Kohler, H. P./Billari, F. C./Ortega, J. A. (2006). Preston, S. H./Hartnett, C. S. (2011) analyze the future of american fertility.

are only temporary as this generation born during the high fertility years pass away. On the other hand, the impacts of increasing life expectancy are of a more permanent nature. The implication of an aging population is an increasing ratio of people in retirement age compared to the working age population. The retirement of the baby boom generation will increase this share as those cohorts retiring will be followed by smaller cohorts. Analogously, improvements in life expectancy directly cause an increase of people in retirement at any given time. Due to the increased ratio of people in retirement relative to people of working age there are serious financial problems for pay-as-you-go financed pensions (mostly public pensions) as well as to funded pensions (generally private pensions)⁴.

Public pension schemes would see the number of young working people contributing dwindle relative to older retirees withdrawing benefits. This would cause increasing public expenditures on pensions. Additionally, many public pension systems are designed in the manner which creates incentives for early retirement and worsens the impacts of demographic changes on public pension expenditures⁵. The aging population has, however, also negative effects on a capital funded pension system since the capital market return will drop due to the demographic change. This result can be explained by two effects:

- The capital intensity increases since the capital funded pension schemes initiate a higher capital accumulation in the whole economy. At the same time, the number of people in

the workforce diminishes due to demographic changes. This leads to a price increase for the factor workforce and to a price decrease for the factor capital. The capital market return will consequently fall.

- A second argument is a predicted collapse of prices on the capital markets. Due to the many retired sellers and the low number of young buyers, the prices of the assets (shares, bonds, real estate etc.) must fall.

In this article, we introduce firstly a macroeconomic model of overlapping generations in which we can analyze the effects of the demographic changes on pension schemes as well as the interactions between pension systems on the one hand and capital markets on the other⁶. The model consists of three building blocks:

- A population model which describes the presumptions with respect to population development,
- Modeling of enterprises in view of the dependence between the existing pension system and the real economic variables such as wage and capital stock and
- Description of risks on financial markets as well as the interactions between pension systems and stock prices or interest rates.

Furthermore, we will examine the effects of pension systems on welfare distribution and consumption as well as the rate of saving within generations.

2. THE MODEL

2.1. Population model

To facilitate the modeling, the model uses discrete time steps $t \in N$. The

⁴Börsch-Supan, A./Ludwig, A (2011) analyze the effects of the aging process in France, Germany and Italy with pay-as-you-go-financed social security systems.

⁵E.g. Duval, R. (2003).

⁶E.g. Gräf, B./Schattenberg, M. (2006); Börsch-Supan, A./Ludwig, A./Winter, J. (2006); Krueger, D./Ludwig, A (2007).

population consists of $J+1$ generations. All consumers live for exactly $J+1$ time steps (overlapping generations model) and exhibit a homogeneous savings and consumption behavior within one generation. Such a model is particularly suited to describe the financial interactions between the generations. Each generation is identified in each period by an index $j=0\dots J$ that describes the remaining life

expectancy. Consequently, $j=J$ is the youngest generation to be born at the beginning of the considered period, $j=0$ represents the oldest generation that dies at the end of the period. The entire population is thus represented by a vector $N_t=(N_t^{(j)})_{j=0}^J$, where $N_t^{(j)}$ is the number of consumers in generation j at the time t . The time evolution of the population is described by the birth rate N with:

$$N_t^{(j)} = N_{t-1}^{(j+1)} \quad j = 0, 1, \dots, J - 1 \tag{1}$$

$$N_t^{(J)} = N(N_{t-1}) \tag{2}$$

Equation (1) describes the aging of the overlapping generations, equation (2) describes the birth rate as a function

$N(N_{t-1})$, depending on the population structure in the period before. In the case of a constant population we find:

$$N(N_{t-1}) = N_{t-1}^{(J)} \Rightarrow N_t^{(j)} = N_{t-1}^{(j)} = N_t^{(j)} \quad \forall t, j = 0, \dots, J. \tag{3}$$

For the numerical simulations in later sections of this work, the demographic change of the population is described by the

following model⁷. Here the population is modeled by age-dependent birth rates $n^{(j)}$:

$$N(N_{t-1}) = \sum_{j=0}^J N_{t-1}^{(j)} n^{(j)} \left(1 + \exp\left(-n_2 \sum_{i=0}^J N_{t-1}^{(i)}\right) \right) \tag{4}$$

Using $n_2=0$ we have constant growth rates and thus an exponential population change. For $n_2>0$ the birth rates decrease as a function of the total population

$\sum_{i=0}^J N_{t-1}^{(i)}$. With the birth rates added up over all generations $n := \sum_{j=0}^J n^{(j)}$ the asymptotic stationary value is obtained

$$\bar{N} := \frac{1}{(J+1)n_2} \ln \frac{n}{1-n}, \tag{5}$$

so that the population tends to the constant distribution $N^* = (\bar{N})_{j=0}^J$.

Furthermore, the model of overlapping generations contains a retirement threshold j_L for the consumers. It is assumed that each member of the

working ($j= j_L, \dots, J$) provides $\bar{L}^{(j)}$ units of labor to the workforce market. The total labor supply of the economy L_t^s is given by:

$$L_t^s := \sum_{j=j_L}^J \bar{L}^{(j)} N_t^{(j)} \tag{6}$$

For simplicity it is assumed that taxes do not exist in the model. A share τ_t of gross wages has to be paid in a pay-as-

you-go system, so that each worker with the gross wage per unit of work ω_t gets the net wage:

⁷E.g. Hillebrand, M. (2008), p. 146 ff.

$$e_t^{(j)} = (1 - \tau_t)\omega_t \bar{L}^{(j)}, \quad j \geq j_L \tag{7}$$

The pension income is composed of total income from work activities, divided by the number of pensioners: the product of the contribution rate and the

$$e_t^{(j)} = \tau_t \frac{\omega_t L_t^s}{\sum_{j=0}^{j_L-1} N_t^{(j)}} \quad j < j_L \tag{8}$$

All individuals can consume their current wealth in a period or transfer it to the next period by two investment opportunities. Using these two alternatives, there is also the possibility to provide for the retirement phase. On the one hand, there is a corporate bond with a maturity of one period available. This may be traded at time t at unit price and attains in the next period $t+1$ one from the company's success dependent return R_t . As R_t is determined at time t , the bond is a risk-free opportunity to transfer wealth to the next period. As an alternative,

an investment in shares is possible. The shares are traded on the rates p_t and deliver in each period a stochastic dividend d_t . In the model short-selling of bonds is possible ("credit"), whereas for shares this is not possible. The portfolio of a consumer $j \in 1, \dots, J$ is described by $z_t^{(j)} := (y_t^{(j)}, x_t^{(j)})$. Here $y_t^{(j)}$ is the number of bonds and $x_t^{(j)}$ is the number of owned shares.

The wealth $w_t^{(j)}$ of a consumer is given as the sum of wage or pension income and the income from capital investment decisions $z_{t-1}^{(j)}$ of the previous period.

$$w_t^{(j)} := \begin{cases} e_t^{(j)}, & j = J \\ e_t^{(j)} + R_{t-1} y_{t-1}^{(j+1)} + x_{t-1}^{(j+1)} (p_t + d_t), & j = 0, 1, \dots, J - 1. \end{cases} \tag{9}$$

It is assumed that all individuals are selfish and do not pass on an inheritance. This implies that consumers in their last period of life sell all shares and do not buy bonds.

2.2. Consumer Behavior

This section describes the decision-making process, which part of their wealth the individuals use for consumption and how they invest the rest in bonds or shares. The goal of the individual here is to maximize their wealth within a planning horizon.

The decisions are influenced by expectations of the consumers regarding their future wage or pension income $\hat{e} = (\hat{e}_1, \dots, \hat{e}_N)$ and the expected bond yield $\hat{R} = (\hat{R}_1, \dots, \hat{R}_{N-1})$ within the planning horizon of N periods. In addition, the decisions are influenced by the uncertainty regarding the impact of unforeseen developments of stock prices and dividends. For the consumer different strategies for consumption c , bond investment y and share investment x are available. Within the planning horizon a strategy has to fulfill the following conditions in each period:

$$c_n + y_n + x_n p_n = w_n \tag{10}$$

$$c_n + y_n + x_n p_n = \hat{e}_n + \hat{R}_{n-1} y_{n-1} + x_{n-1} (p_n + d_n) \quad n = 1, \dots, N \tag{11}$$

This must be met for all possible (random) stock prices p_n and dividend payments d_n . The existence of such a strategy under certain constraints, which

will not be dealt with here, is mathematically proven⁸. The goal of a consumer is to maximize the logarithmic utility function that reaches to the end of his life:

$$U^{(j)} = \ln(c_0) + \sum_{n=1}^j \beta^n \ln(c_n). \quad (12)$$

The discount rate β represents the general fact that individuals prefer consumption in the present compared to consumption in the future.

It has been found that it is not necessary to consider the entire planning horizon in the model⁹. The decision problem can be traced back to a single-stage problem, so that demand functions are specified which describe the optimized consumer demand for one period. The

derivation of the formula models the uncertainty of the stock market using a probability distribution v_q for the sum $q=p+d$ of share price and dividend. The expectation value μ of the distribution is chosen to fit the next-period consumer expectation for stock price plus dividend. The width of the distribution is determined by the variance Σ , the normalization of the probability distribution is guaranteed by ξ .

$$v_q = \frac{1}{\xi} \exp\left(-\frac{(q-\mu)^2}{2\Sigma}\right) \quad (13)$$

Thus we get the following expressions for the benefit optimized values for tendency for consumption $\varphi_{c,t}^{(j)}$

, share demand $\varphi_{x,t}^{(j)}$ and bond demand $\varphi_{y,t}^{(j)}$ for each generation¹⁰.

$$\varphi_c = \bar{c}(w + \hat{\varepsilon}_0/R) \quad (14)$$

$$\varphi_x = (1 - \bar{c})(w + \hat{\varepsilon}_0/R)\theta(Rp, v_q)/p \quad (15)$$

$$\varphi_y = (1 - \bar{c})(w + \hat{\varepsilon}_0/R)(1 - \theta(Rp, v_q)) - \hat{\varepsilon}_0/R, \quad (16)$$

where $\hat{\varepsilon}_0$ is the discounted expected future income:

$$\hat{\varepsilon}_0 := \hat{\varepsilon}_1 + \frac{\hat{\varepsilon}_2}{R_1} + \dots + \frac{\hat{\varepsilon}_j}{R_1 \dots R_{j-1}} \quad (17)$$

$$\bar{c} = \frac{1}{1 + \beta + \dots + \beta^j} \quad (18)$$

$$\theta(Rp, v_q) = \arg \max_g \left(\int \ln(Rp + g(q - \pi)) v_q dq \right) \quad (19)$$

The functions φ provide the optimum with respect to tendency of

consumption, investment in shares or bonds, as a function of the prices R and

⁸Hillebrand, M. (2008), p. 15 ff.

⁹Hillebrand, M. (2008), p. 20 ff.

¹⁰The indices j and t are for clarity not explicitly specified in the following.

p and wealth w , as well as expectations $\hat{\epsilon}_0$ and μ . The optimal consumption is a fraction \bar{c} of the sum of current wealth w and the discounted expected income $\hat{\epsilon}_0/R$. This amount is referred to as lifetime earnings. The percentage of consumption only depends on the discount factor β and is independent from expectations of future financial market prices¹¹.

For the consumer only the wealth w is directly available. The quantity $\hat{\epsilon}_0/R$ has to be compensated by short-selling of bonds. For this reason the term $\hat{\epsilon}_0/R$ appears in the function for the bond demand. The

optimum volume of shares is determined by the variable θ , which is only influenced by present (R,p) and via v_q by expected (μ,Σ) financial market prices.

In order to get a quantitative evaluation of the model, a concrete behavior of how the consumers form their expectations regarding the financial market variables has to be assumed. For consumer expectations the following assumptions are made: Each consumer assumes to achieve in a future period the income that a consumer of the appropriate age, in the current period receives:

$$\hat{\epsilon}_{t,t+n}^{(j)} = e_t^{(j-n)}, \quad n = 1, \dots, j, \quad j = 1, \dots, J \tag{20}$$

Regarding the bond yield consumers expect stability of the current value.

$$\hat{R}_{t,t+n} = R_{t-1}, \quad n = 1, \dots, J-1 \tag{21}$$

For the expectation of future stock prices including dividends μ_p , the consumer corrects in each period, its previous

expectations μ_{t-2} , using an error correction by means of a weighted (ρ) prediction error $q_{t-1} - \mu_{t-2}$:

$$\mu_t = \mu_{t-2} + \rho(q_{t-1} - \mu_{t-2}) \quad 0 \leq \rho \leq 1 \tag{22}$$

2.3. Model of Company Decisions

In this section, the production and investment behavior of the business side is modeled. For simplicity, only one company is considered. In each period t the firm uses

its capital stock K_t and labor L_t to produce. The production process in the form of the production function F is exposed to random environmental influences η_t :

$$F(L_t, K_t, \eta_t) = \kappa L_t^\alpha K_t^{1-\alpha} + \eta_t, \quad \kappa > 0, \alpha \in [0,1] \tag{23}$$

The time evolution of the capital stock is determined by the depreciation

rate δ and the investment I_t :

$$K_{t+1} = I_t + (1-\delta)K_t. \tag{24}$$

The entrepreneurial investment decision I_t is financed only by issuing a number of bonds B_t at unit price. In the following it is assumed that there is no proportionality between capital stock increase I_t and the collected amount of

money B_t .¹² Instead, it is assumed that the larger the desired ratio of investment I_t to existing capital stock K_t , the more money must be taken in order to achieve the desired capital stock increase. The link is called ‘‘adjustment cost function’’.

¹¹This is a special property of the logarithmic utility function in equation (12).

¹²E.g. Abel, A. B. (2003) or Altig, D. et al. (2001).

$$B_t = 0 \qquad I_t = 0 \qquad (25)$$

$$B_t = K_t \gamma_0 \exp(\gamma_1 I_t / K_t) \quad I_t > 0, \gamma_0 > 0, \gamma_1 > 1/\delta \qquad (26)$$

This function is discontinuous at $I_t=0$. This reflects the fact that in case of no investment no money is needed, while at a minimal investment already fixed costs $K_t \gamma_0$ accrue.

The bonds that have been issued in period t in order to finance the investment decision generate the obligation to pay

in period $t+1$ the sum $R_t B_t$ to the investor. In addition to paying off the bonds, the company has to pay the gross wages. It is assumed that the remaining money will be fully distributed as dividends to shareholders. The dividend per share is therefore (where \bar{x} is the total number of shares):

$$d_t = \frac{F(L_t, K_t, \eta_t) - \omega_t L_t - R_{t-1} B_{t-1}}{\bar{x}} \qquad (27)$$

How does the company make decisions? What amount of labor is needed? How much should be invested? As part of the model it is believed that these decisions are made under the constraint of maximum dividend payments. On the one hand, the demand for a maximum dividend payout in the current period, determines the labor demand in this period, on the other hand, the desire to maximize the dividend

of the subsequent period determines the investment decision. In detail: At a decision point, the company has a capital stock K_t , knows the environmental impact η_t and the bond repayment $R_{t-1} B_{t-1}$ from the investment decision in the previous period. The gross wage ω_t is treated here as a parameter. By partial derivation of (27) with respect to L_t an extremum of d_t for the labor demand at a given wage level ω_t can be found.

$$L_t = \left(\frac{\alpha \kappa}{\omega_t} \right)^{\frac{1}{1-\alpha}} K_t \qquad (28)$$

For the investment decision it is assumed that the dividend in the next period.

$$d_{t+1} = \frac{F(L_{t+1}, K_{t+1}, \eta_{t+1}) - \hat{\omega}_{t+1} L_{t+1} - R_t B_t}{\bar{x}} \qquad (29)$$

should be maximized. Here $\hat{\omega}_{t+1}$ is the expected wage level of the next period. With K_{t+1} from equation (24) and partial

derivative $\partial d / \partial I$, zero-setting and using equation (28) follows for the optimal investment level:

$$I = \frac{1}{\gamma_1} \ln \left(\frac{(1-\alpha)\kappa}{\gamma_0 \gamma_1 R} \left(\frac{\alpha \kappa}{\hat{\omega}_{t+1}} \right)^{\frac{\alpha}{1-\alpha}} \right) K_t \qquad (30)$$

This results in the bond offer

$$B = \frac{(1-\alpha)\kappa}{\gamma_1 R} \left(\frac{\alpha \kappa}{\hat{\omega}_{t+1}} \right)^{\frac{\alpha}{1-\alpha}} K_t \qquad (31)$$

For the specific determination of wage expectations $\hat{\omega}_{t+1}$ different models can be used. In this work, the assumption of constant wages is used:

$$\hat{\omega}_{t+1} = \omega_t \tag{32}$$

2.4. Time Evolution of the Model and Formation of Equilibrium Price

Using the models derived in the previous sections for behavior of consumers and companies, we can now determine the market equilibrium. Within the sequential structure of the model this is done at a time t by means of five steps. From the previous period $t-1$ the population N_{t-1} , capital stock K_{t-1} , company investment I_{t-1} , share and bond investment of consumers $z_{t-1} = (x_{t-1}^{(j)}, y_{t-1}^{(j)})_{j=1}^J$, bond volume B_{t-1} and bond yield R_{t-1} are given.

Step 1: At the beginning of the period the development of the population takes

place. The population N_t develops from N_{t-1} according to the model from section 3.1. It follows the labor supply during the current period according to equation (6). The capital stock of the current period is given by the capital stock and investments of the previous period from equation (24).

Step 2: Using the labor demand function of the company according to equation (28), the labor supply equation (6) and the assumption of a cleared labor market ($L_t^s = L_t^p$, full employment) it is possible to derive the equilibrium gross wage ω_t of the period t :

$$\omega_t = \kappa\alpha \left(\frac{K_t}{L_t^s} \right)^{1-\alpha} \tag{33}$$

Depending on the preset contribution rate τ of the pay-as-you-go system, the income of the consumers without investment income follows according to equations (7) and (8). According to the effect of random production shock η_t the dividend payment d_t for the current period is determined using equation (27).

Step 3: Based on these results each consumer forms the expectations for the next periods. For the income equation (20) is valid, for the expected bond yield (21) is used and the expectations for the

stock prices follow equation (22). The companies make predictions on the wages of the next period according to equation (32). Based on these expectations the consumers form their demand behavior according to equations (14) - (16). The company determines the bond volume according to equation (31).

Step 4: From the previous steps result share and bond demand and bond offering. The prices are obtained from market clearing conditions. These are for the stock and bond market:

$$\sum_{j=1}^J N_t^{(j)} \varphi_x^{(j)} = \bar{x} \tag{34}$$

$$\sum_{j=1}^J N_t^{(j)} \varphi_y^{(j)} = B_t \tag{35}$$

Therefore we get for the actual period t the bond yield R_t and the stock price P_t .

$$R_t := \frac{B + \sum_{j=1}^J N_t^{(j)} \bar{c}^{(j)} \hat{\varepsilon}_{0,t}^{(j)} + \pi_t \sum_{j=0}^{J-1} N_t^{(j)} \bar{c}^{(j)} x_{t-1}^{(j+1)}}{\sum_{j=1}^J N_t^{(j)} (1 - \bar{c}^{(j)}) e_t^{(j)} + \sum_{j=1}^{J-1} N_t^{(j)} (1 - \bar{c}^{(j)}) [R_{t-1} x_{t-1}^{(j+1)} + d_t x_{t-1}^{(j+1)}]}$$

$$p_t := \frac{\pi_t}{R_t} \tag{36}$$

With

$$\pi_t = \mu_t - v_t(\mu_t, \Sigma_t, N_t, K_t, \hat{e}_t, \hat{R}_t, \hat{\omega}_t) \Sigma \bar{x} \tag{37}$$

Here π_t is an adoption of the expectation μ_t to reality. The correction depends on the function v_t which results from two nested implicit equations, which are listed here without derivation¹³.

$$1 - \frac{\lambda^*(\mu_t - v_t \Sigma \bar{x}, \mu_t, \Sigma)}{\bar{x} \sqrt{\Sigma}} \left(\mu_t \bar{x} - v_t \Sigma \bar{x}^2 + \sum_{j=1}^J N_t^{(j)} \hat{\varepsilon}_{0,t}^{(j)} + B_t \right) = 0 \tag{38}$$

The auxiliary value λ^* is derived from the solution of the following equation, where the finite limit of integration $\bar{\varepsilon}$ is chosen such that negative q -values are prevented

$$\int_{-\bar{\varepsilon}}^{\bar{\varepsilon}} \frac{\frac{\mu_t - \pi_t}{\sqrt{\Sigma}} + \varepsilon}{1 + \frac{\mu_t - \pi_t}{\sqrt{\Sigma}} \lambda + \lambda \varepsilon} v_\varepsilon d\varepsilon = 0 \tag{39}$$

For the previously determined prices, the consumer demand for stocks and bonds is satisfied and it is thus:

$$x_t^{(j)} = \varphi_x^{(j)}, \quad j = 1, \dots, J \tag{40}$$

$$y_t^{(j)} = \varphi_y^{(j)}, \quad j = 1, \dots, J \tag{41}$$

$$B_t = B \tag{42}$$

Step 5: Finally, the individuals make their consumption decision and the company uses the money obtained by issuing bonds to fund the investments:

$$c_t^j = \varphi_c^{(j)}, \quad j = 0, 1, \dots, J \tag{43}$$

$$I_t = I \tag{44}$$

With this procedure, the development of the model is set within a period. For the next period in time the sequence begins again.

2.5. Calibration of the Model

In this section the parameters for the simulations are presented and motivated. The free parameters shall approximate

¹³Hillebrand, M. (2008), p. 67 ff.

reality. The values that are used have been derived from empirical studies.

For the simulations with the OLG-model at first a constant population with $N_t^{(j)} = \bar{N} = 1000$ consumers is assumed. Afterwards a dynamic population is studied. For life expectancy we use in both chapters $J=14$ and the retirement age is set to $j_L=6$. Consequently, always 15 generations live at the same time. Thereof nine go to work and six are retired. Assuming that an individual consumes during 60 years (a childhood phase without consumption does not exist in the model), then a time step in the model corresponds to four years of life. Any consumer who is not already in retirement, provides one unit $\bar{L}^j = 1$ to the labor market. The discount rate β , that is relevant for the optimization of the consumers, is taken from an empirical study¹⁴. This results in a discount rate of 0.011 corresponding to an annual discount factor of $1/1.011$. With a time basis of four years $\beta = (1/1.011)^4 \approx 0.96$ is obtained.

The constant parameters of the stock price expectancy are set to values of $\bar{\varepsilon} = 0.92$ for the maximum share price variation and $\Sigma = 0.97$ is the width of the Gaussian distribution. The parameter ρ , that describes the temporal adjustment of the expectation value μ_t is set to $\rho=0.5$. For the parameter α in the production function empirical studies found values between 0.6 and 0.7. For the simulations a value of $\alpha = 0.66$ is used. The parameter κ in the

production function serves essentially as a scaling factor and is set to the value $\kappa=2.5$. The average annual depreciation rate in Germany between 1960 and 1990 was approximately 0.0528¹⁵. Within the four year period of the simulation step $\delta = 1 - (1 - 0.0528)^4 \approx 0.2$ is therefore used. For the adjustment cost function $\gamma_0=0.02$ and $\gamma_1=7.5$ is used, in order to fit empirical studies¹⁶ in form of equation (26). As a size for the production shocks η uniformly distributed random numbers between 0 and 2000 were used. As initial values we use for the capital stock $K_0=11500$, for stock prices including dividends $q_0=14,5$ and for bond yield $R_0=1,14$. Stock and bond volume are initially equally distributed across all generations.

3. SIMULATION RESULTS

In this section simulation results are presented which were determined using the previously described model. At first some basic properties of the model were demonstrated using a constant population. Then, the interactions between the pension system and an aging population are analyzed.

3.1. Results for a Constant Population

First of all, the possibilities of the model are shown for the case of a stationary population. The contribution rate is set to the value $\tau = 0.1$. The implementation of the simulation code was done using MATLAB.

¹⁴E.g. Hurd, M. D. (1989), p. 801.

¹⁵E.g. Börsch-Supan, A. et al. (2003), p. 167.

¹⁶E.g. Abel, A. B. (2003) and Altig, D. et al. (2001).

Figure 1. Time evolution of the simulation parameters for $\tau = 0.1$ and a constant population

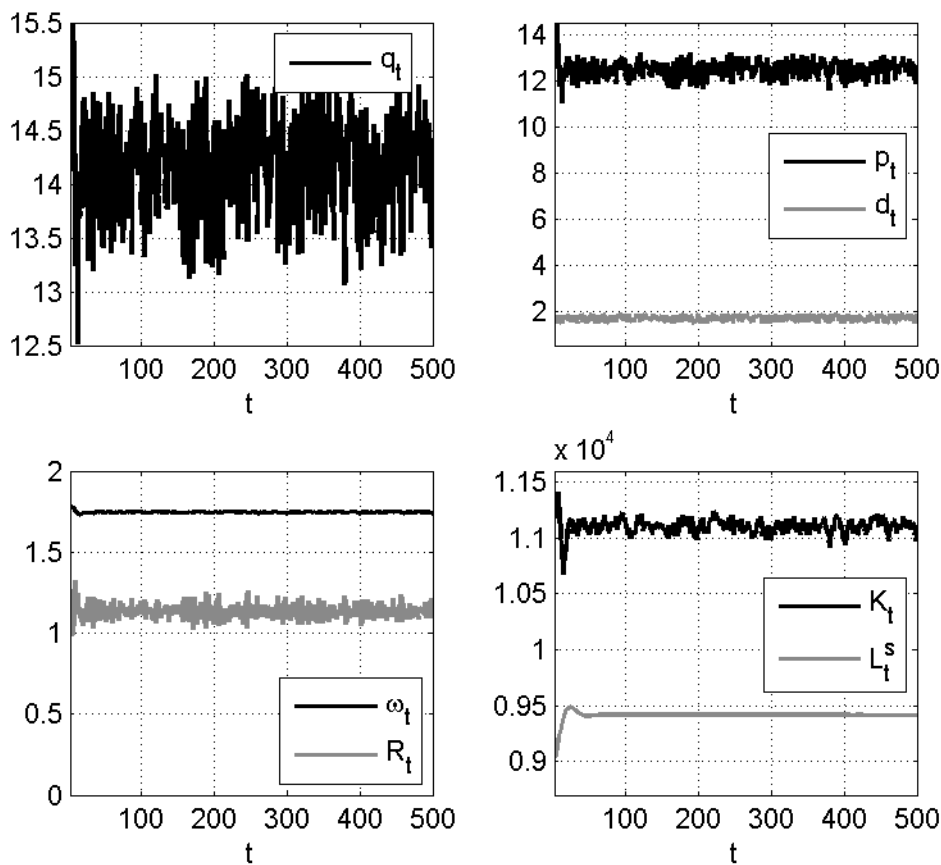
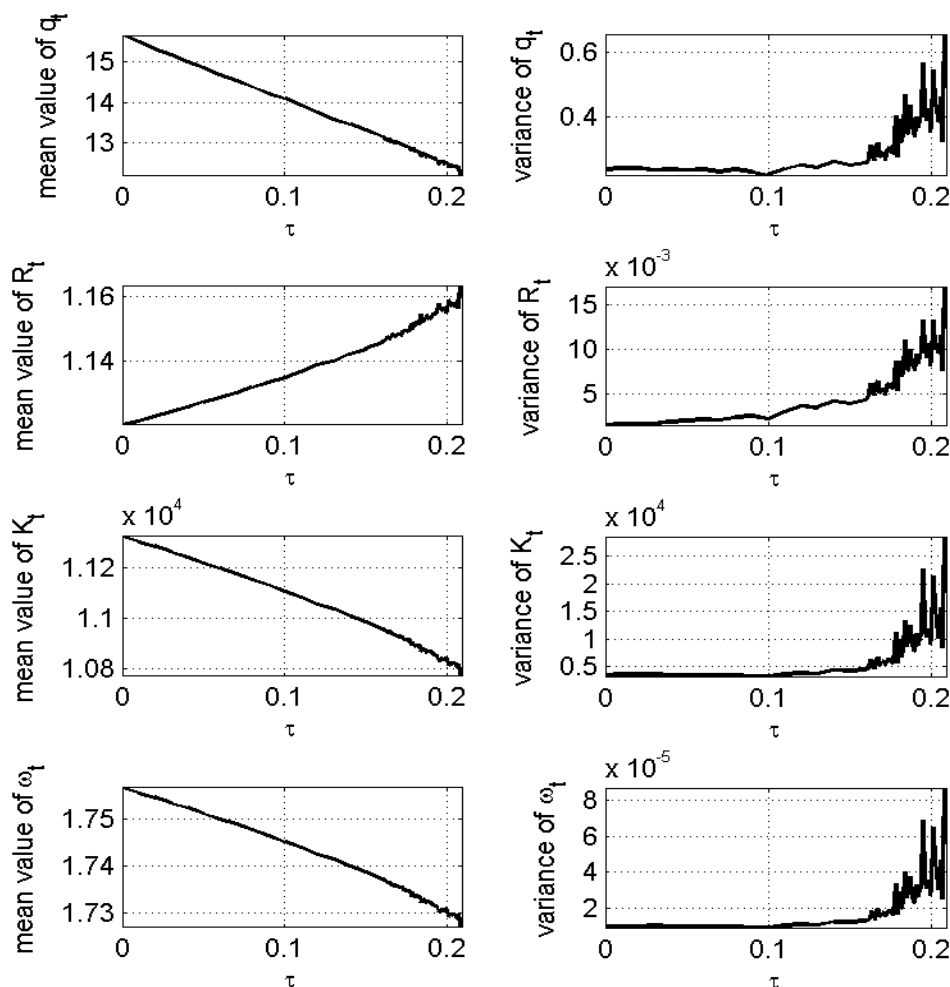


Figure 1 shows the time evolution of the model with the previously calibrated parameters and the presented population model. It reaches its equilibrium state already at $t=50$, resulting in a constant labor supply L_t^s . Therefore the figure shows for $t>50$ the dynamics of the other parameters for the case of a constant population. The figure also serves to verify the consistency of the simulation. For $t>50$ all parameters fluctuate around a fixed value. It turns out that the wages are very stable compared

to the financial market variables. The fluctuations in stock prices including dividends q_t are essentially caused by the fluctuations of the stock prices themselves and to a small extent by the volatility of dividends. We also find, as expected, a positive correlation between stock prices and dividend payments. The bond yield R_t fluctuates around a value of 1.135, which corresponds to an annual interest rate of about 3%. This is consistent with the empirical expectation.

Figure 2. Dependence on the contribution rate τ for the average values and standard deviations.



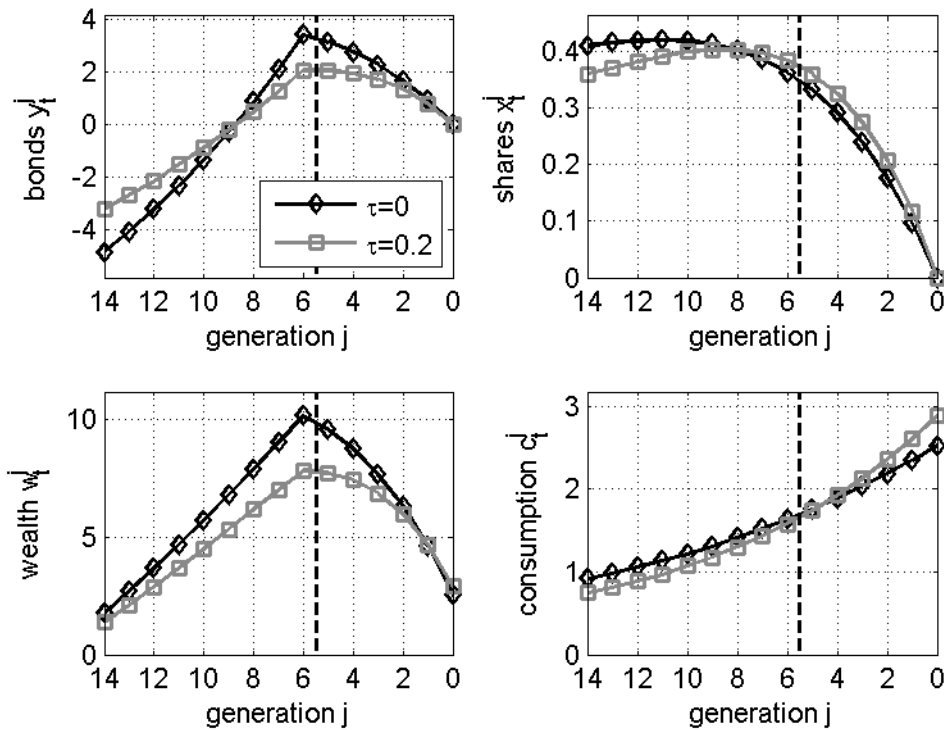
In order to investigate the influence of the pension system on the variables in the following the contribution rate τ is varied (Figure 2). It is immediately obvious that when decreasing the contribution rate the capital stock increases. The basic assumption, that a funded system is superior to a pay-as-you-go system is depicted in the model. In addition, it can be seen that the average for the stock price including dividends decreases with an increasing contribution rate, whereas the bond yield increases.

Furthermore the variability around the time average (characterized by the standard deviation) increases significantly with increasing contribution rates. It can

also be recognized that the model breaks down at a value of $\tau > 0.21$. Here the model fails because consumers go bankrupt. A more detailed analysis of the strong fluctuations shows that they are caused by fluctuations of the stock prices and not by the dividends. The wage level is largely independent of the contribution rate. Here also hardly any impact of the large fluctuations of the financial market variables for large contribution rates can be found.

Figure 3 shows the steady state distribution across generations for the variables y (bonds), x (shares), w (wealth) and c (consumption).

Figure 3. Volume of bonds, shares, wealth and consumption across generations



It is striking that young consumers secure credit by short selling of bonds. The borrowing is reduced with time, until for $j < 9$ the bonds become an investment opportunity. The high-risk form of investment in the stock market is used more specifically in young years, the shares are sold with age. This appears more strongly in the absence of a pay-as-you-go system ($\tau = 0$). In this case the wealth is for all times larger than with a pay-as-you-go system. The consumers successfully compensate the missing pension income in retirement age by additional capital income. The

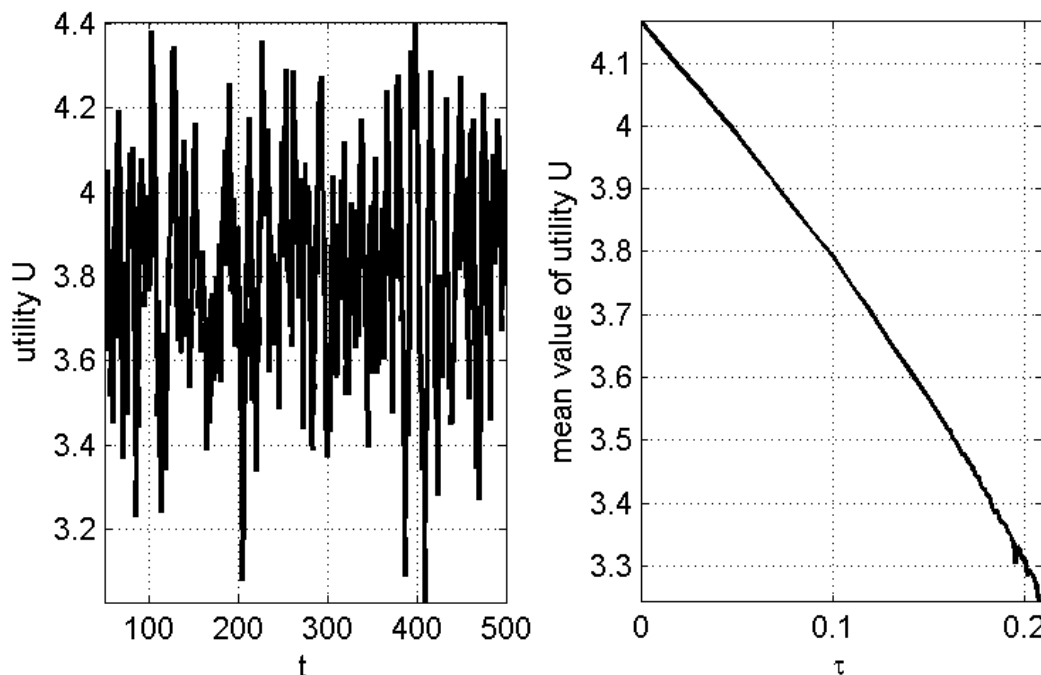
wealth reaches in both cases the maximum value just before the end of the working period. The consumption, however, rises in both cases monotonically with increasing age, where the increase for $\tau = 0.2$ is steeper. This shows that without a pay-as-you-go system, consumption moves forward, and is even distributed more homogeneously over the many stages of life.

To assess which pension system is beneficial for the population, the lifetime welfare in the form of the following dimensionless variable is examined:

$$U_i(\tau) = \sum_{j=0}^J \beta^{J-j} \ln c_{t-j}^{(j)} \tag{45}$$

For each time $t > J$ the variable describes the welfare over the lifetime of a consumer belonging to generation J , who

was born in period $t - J$ and will die at the end of period t . In figure 4, this welfare function is shown.

Figure 4. Welfare in case of a constant population for different contribution rates.

For a stationary population, a higher contribution rate leads to a reduction of the lifetime welfare. Consequently, for a constant population the introduction of a pay-as-you-go system is not advisable. In an existing system, the contribution rates should be reduced in order to achieve a long-term gain of welfare for all generations. In the long run an existing pay-as-you-go system should be abolished. The problem is that following a reduction of contribution rates, the retired generation at this time suffers from welfare losses. It is important therefore, to make the changeover as gently as possible.

3.2. Results for an Aging Population

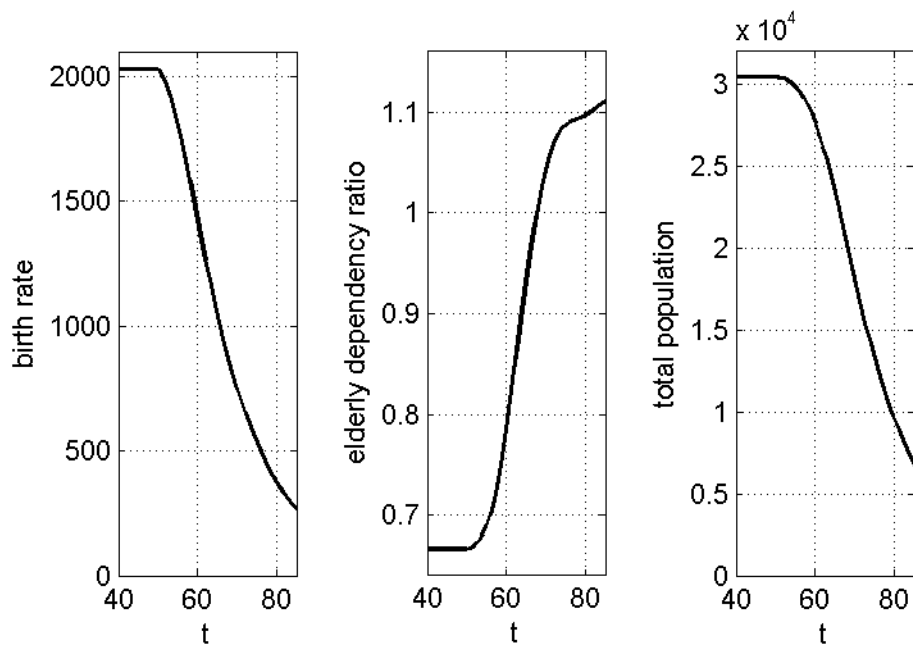
In this section we examine how the model responds to changes in population structure. The German population will age dramatically in the coming years. For a pension system the relevant parameter is the ratio of the number of contributors to the number of recipients (economic dependency ratio). There is a difference if we look at the entire population. This is

based on the fact that not all young people are actually contributors (unemployment, employment inclination). Börsch-Supan¹⁷ investigated the transfer of the elderly dependency ratio into predictions of the economic dependency ratio. It is essentially a scaled course of the elderly dependency ratio, with an increase of 66% in 2010 to one (scenario-dependent) rise up on a range of > 100% by 2030.

In the model discussed here, all consumers are also contributors, thus both parameters coincide. To simulate the aging curve, the birth development shown in figure 8 was used. For $t < 50$ the population is constant. This initial time phase is used to bring the model to its steady state. At $t = 50$ the onset of a demographic change is modeled by reducing the generation dependent birth rates $n^{(i)}$ in each period by 1.4%. For $t > 70$ at each time step a reduced adjustment of the birth rates is applied in order to model the slower and almost linear increase of the elderly dependency ratio.

¹⁷Börsch-Supan, A et al.. (2003), p. 158.

Figure 5. Population dynamics in the model



This results in the dependency ratio shown in the center graph of figure 5. This model is close to the forecast. The development of the total population shown in the right graph is not very realistic. This is because part of the aging is due to an increase in life expectancy, while in the model all individuals always reach the same age. The aging therefore had to be fully modeled by changing the birth rates. In addition, the aging of the population was slowed in order not to overstretch the model (1 time step = 4 years).

Figure 6 and 7 show the time evolution of the model parameters for an aging population with contribution rates $\tau = 0$ and $\tau = 0.2$. Most striking is the drastic decline in stock prices and capital stock. Both variables are dramatically reduced. The drop is largely independent of the contribution rate and in the magnitude comparable to the decrease in the total population. This decline in stock prices is known as the phenomenon of „asset market meltdown“ and has been predicted by several models¹⁸. Wages are rising slightly due to the scarcity of the labor supply. Also the bond yield increases slightly.

¹⁸ Abel, A. B. (2001), p. 589 ff.

Figure 6. Parameter dynamics for $\tau = 0$ in presence of aging

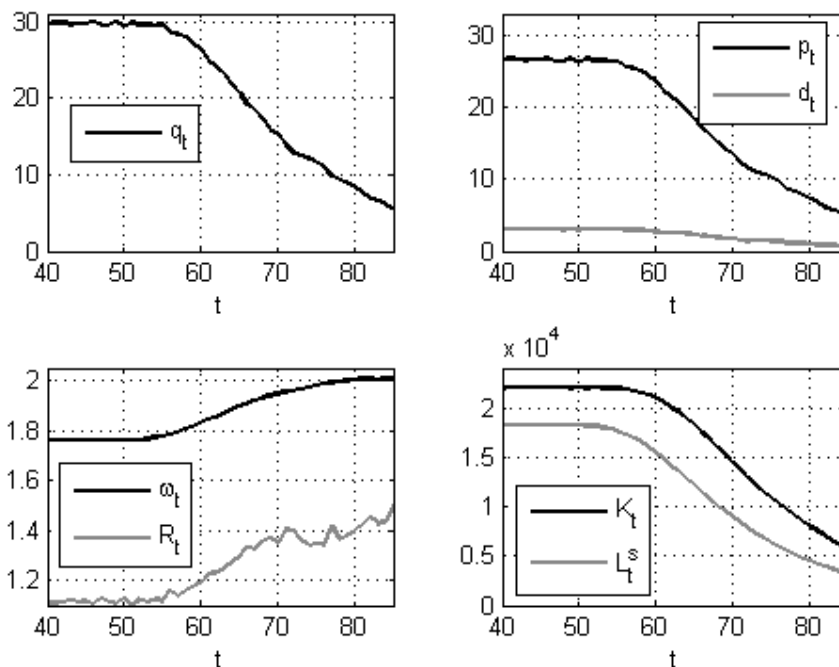
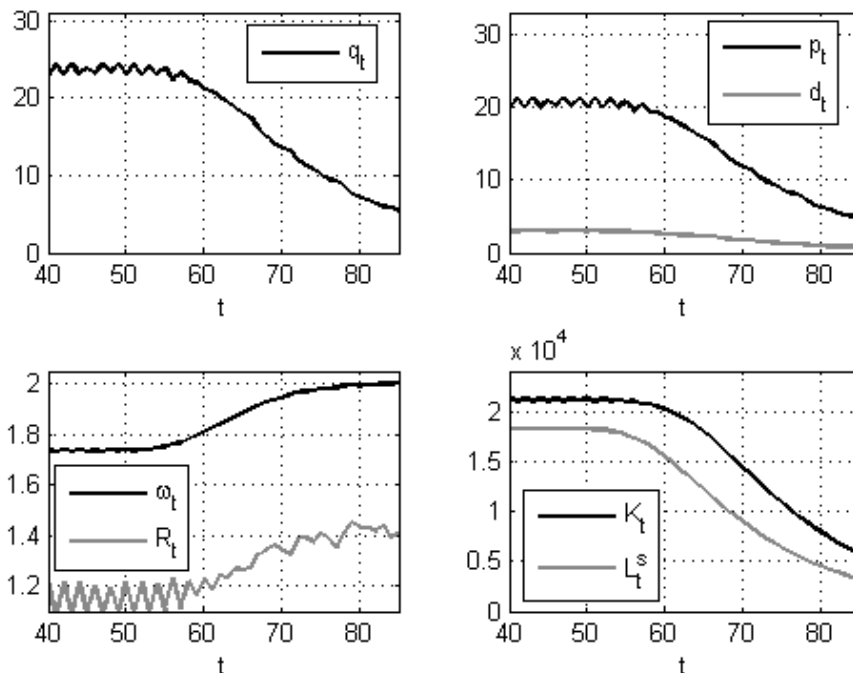
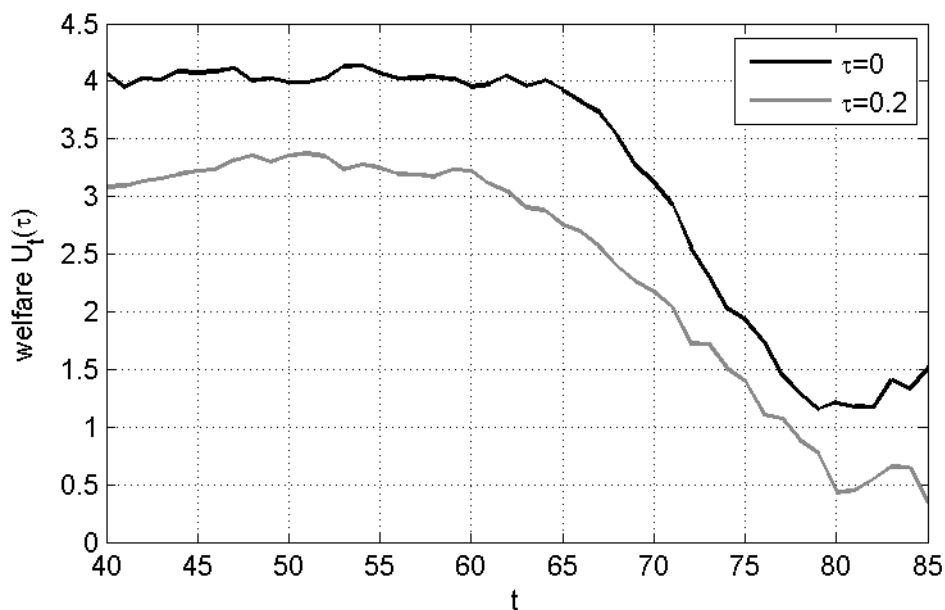


Figure 7. Parameter dynamics for $\tau = 0,2$ in presence of aging



The impact on the welfare of consumers is shown in figure 8 as the average of three different realizations of the

random process in the production function. The curves follow the dependency ratio with a slight delay.

Figure 8. Welfare for constant contribution rate and demographic change

The decline is independent of the contribution rate. For $\tau = 0$ the decline is reduced and the welfare is always larger compared to the case of $\tau = 0.2$. It turns out that demographic change with constant contribution rates will lead to drastic losses in consumer welfare.

3.3. Increasing the Contribution Rates

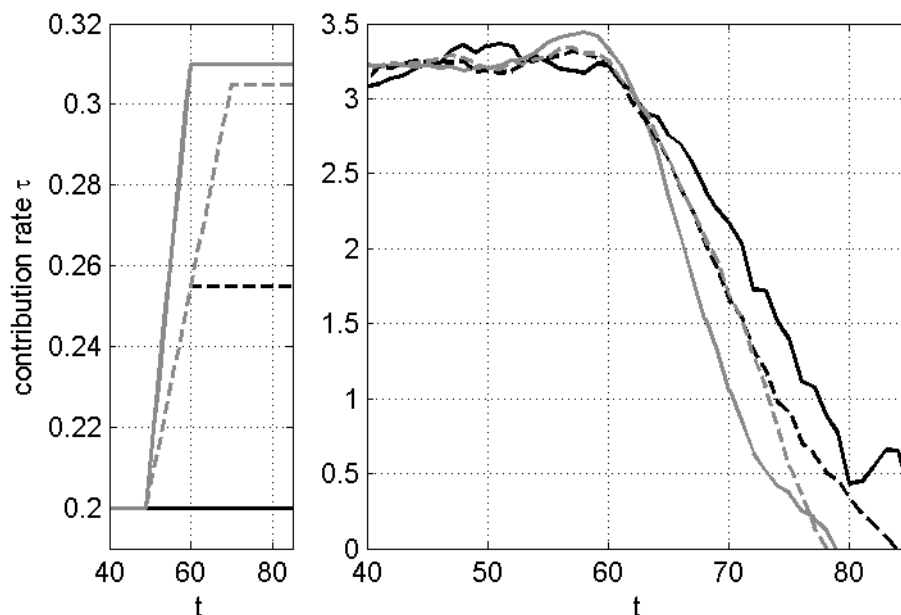
The decline of welfare, stock prices and capital stock as a result of demographic change raises the question whether it is possible to adapt the pension system in order to prevent or reduce these negative effects. In the following sections an existing pension system with a contribution rate of $\tau = 0.2$ and a retirement threshold of $j_L = 6$ is assumed.

One way to overcome this problem is to increase the contribution rate. Because

fewer workers are facing more pensioners a higher contribution rate is needed in order to keep the pension level constant. In Germany, this is discussed in the form of the *Riester/Rürup* formula. In this approach, depending on the evolution of the salaries, the contribution rate in the two previous periods and the population growth, the contribution rate is increased. The dependence of consumer welfare by different variants of the formula was tested using the OLG model presented here already¹⁹. The results achieved there are consistent with those achieved in the following using a simplified linear increase of the contribution rate. As the upper limit a value of $\tau = 0.31$ is assumed, because higher contributions are not politically feasible. In figure 9 the development of welfare is shown for different contribution scenarios.

¹⁹Hillebrand, M. (2008), p. 159.

Figure 9. Welfare in case of demographic change and increasing contribution rates

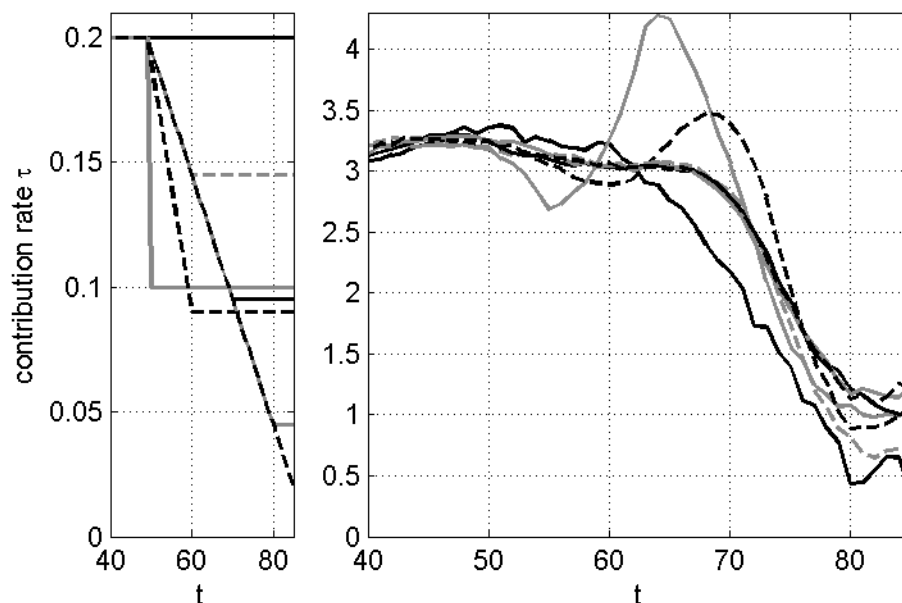


It is found that for all cases after the onset of demographic change at $t = 50$ the level of welfare can be stabilized at first. The level is sometimes even higher than the reference ($\tau = 0.2 = \text{const}$). The initial gain is bought dearly by a sharp drop in welfare later. Starting at $t = 63$ all welfare curves with increasing contribution rates fall below the reference curve and stay there. An increase in the contribution rate is therefore not an appropriate means of preventing or dampening the loss of welfare. The welfare loss is delayed at first but will be more severe later.

3.4. Decreasing the Contribution Rates

As an alternative to an increase in contributions, in this section a contribution reduction or gradual abolition of the pay-as-you-go system is examined as a way to reduce the effects of demographic change. In figure 4 it was shown that for a constant population a lower rate of contribution leads to welfare benefits. It therefore seems quite reasonable to use this potential welfare gain by lowering contribution to compensate for the loss caused by demographic effects. This is shown in figure 10 for different contribution reduction scenarios.

Figure 10. Welfare in case of demographic change and decreasing contribution rates

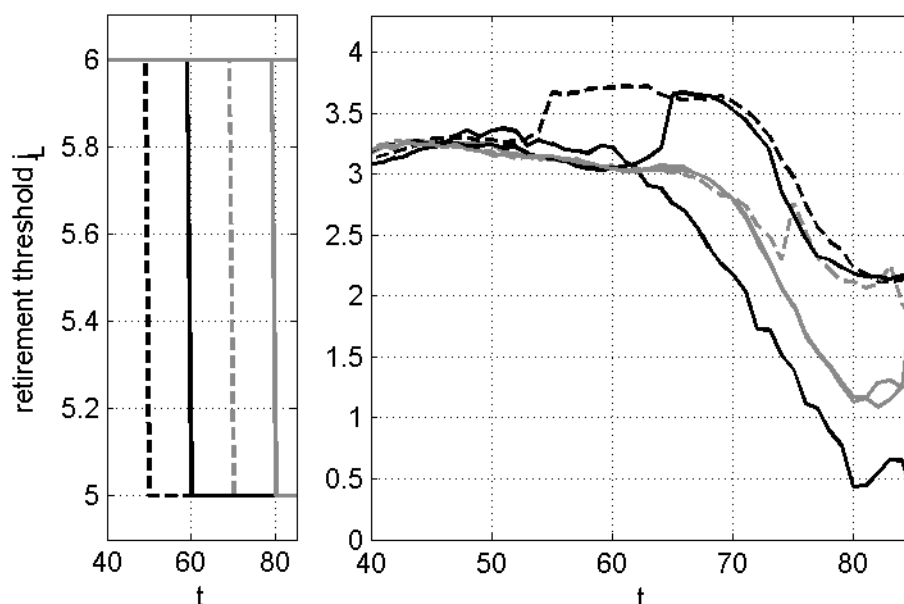


For a very abrupt reduction of the contribution rate a reduction of welfare at $t = 50$ occurs, this is the result of the fact that these generations suffer from reduced pensions, but could not benefit from the low contribution rates during their working life. Afterwards for $t > 60$ these cases show an increase of the welfare to values above the reference curve. This is due to the generations that benefit from the reduced contributions during their working period. This overshoot can be dampened when we pass to a slow reduction of the contributions. For all scenarios, finally the demographic effect prevails and results in a sharp decline in welfare. For all cases we find values for the welfare that lie after $t = 62$ permanently above the reference curve. A reduction in contributions can therefore not prevent the loss of welfare. But it is possible to reduce the strength and to shift in time the decline of welfare.

3.5. Raising the Retirement Age

The analysis so far has dealt only with the contribution rate adjustments to adapt the pension system to demographic development. If we assume that the retirement age is not determined by the physical abilities of workers but is determined by policy, this limit can be considered as a variable parameter. Furthermore, this is the only reform option that directly attacks the elderly dependency ratio by increasing the number of workers and reducing the number of pensioners. In previous studies a value of $j_L = 6$ has been consistently used. In figure 14, the evolution of welfare for different time periods for the raising of the retirement age from $j_L = 6$ to $j_L = 5$ is shown. In all cases the contribution rate is decreased linearly between $t = 50$ and $t=89$ from $\tau = 0,2$ to $\tau = 0$.

Figure 11. Welfare in case of raising the retirement threshold and reduction of contribution rates



Increasing the retirement age leads in the case of an early increase to a leap in the evolution of the welfare. This is due to the generations who receive an additional period of working income instead of a pension. Also, the pension of the older generations is increased. Nevertheless, the demographic effect leads to a decline in welfare. Compared to the scenarios with constant retirement age we find the possibility to reduce the decline of welfare. It is possible to get closer to the goal of a constant welfare if the retirement threshold is increased at a time around $t = 70$. Here the jump in welfare counteracts the demographic effect leading to a welfare evolution that is still decreasing but is much more stable than in previous cases. To ultimately find a welfare curve which is as flat as possible shorter time steps should be used.

4. CONCLUSION

The impact of an aging population is broader than its financial impact on public budgets within a pay-as-you-go pension scheme. Demographic changes will affect the entire economy by affecting productivity, savings, consumption,

interest rates and asset valuations. In the context of the introduced macroeconomic model, we have shown a drastic fall of capital market returns due to an aging population. Therefore, a capital funded pension system is also influenced negatively by demographic changes. Moreover, we examined which effect the demographic changes can have on the welfare of the individuals within a pay-as-you-go pension system. The increasing aging of the population leads here to heavy falls in individual welfare. Different reform variants of the pension system were examined.

An increase of pension age at simultaneous lowering of the contributions and gradual abolition of the financed pension system proves to be the best possibility not to remove but to reduce the welfare loss of the consumers due to demographic changes as far as possible. On the other hand, a policy with increases in contributions as a consequence of the demographic change shows the heaviest welfare losses. Taken into account the recent pension reforms in Germany

(increase of regular retirement age from 65 to 67, introduction of a sustainability factor to limit the increase of contribution), a stronger increase in the retirement age or a faster transition from a pay-as-you-go pension system to a capital funded one would make sense from our point of view. But with an aging electorate, it is questionable whether such a policy will be enforceable²⁰.

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²⁰E.g. *Shoven, J. B./Goda, G. S.* (2011)

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