A TWO-DIMENSIONAL QUASI MODEL FOR SIMULATING FLOW IN OPEN-CHANNELS

Chien Pham Van[1]

Abstract: *This paper proposes a 2D-quasi model for simulations of the lateral distribution of flow velocity and of unit water discharge in open-channel sections. The latters are obtained by solving the governing equation, which is derived from the Reynolds equations and allows for taking into account the gravity, bed shear stress, and turbulent diffusion force in calculations. Using the experimental data, a sensitivity analysis of modeling parameters, e.g. Manning coefficient and eddy viscosity was performed firstly. Approximate values of modeling parameters were then calibrated before the model was validated. Next, the proposed 2D-quasi model was applied to represent the flow of three experimental data sets. Four error estimates were computed to quantitatively assess the quality of simulations, revealing that (i) a good agreement between simulations and observations was obtained and (ii) the proposed 2D-quasi model was successfully used to reproduce flow of all experimental data sets using in the study. The capability of proposed 2D-quasi model was also discussed.*

Keywords: Compound open-channel, 2D-quasi model, flow velocity, eddy viscosity.

1. INTRODUCTION

Natural and restored waterways are comprised of neither solely simple cross sectional areas nor one of basic geometric shapes like rectangles, trapezoids, or simple curved shapes. They are mostly composed of a main channel and one or more adjacent floodplains. Such channels are known as compound channels whose cross-sections are made up of more than one of basic geometric shapes. Generally, water entirely remains in the main channel during low and/or normal flows while water fills the entire main channel and proceeds to spill over into floodplains or overbank areas during high flows. Thus, flow characteristics such as flow velocity and water depth can vary significantly across the section because of the variation of (i) geometry, (ii) bed friction, and (iii) transverse transfer of momentum between the fast flow in main channel and the adjacent slower flow in floodplains (Sellin, 1964).

Besides measurements, numerical models are also applied to study the complexity of flow characteristics in general and in particular in compound open-channels without excessive simplification of the physical processes resolved by models. Among different models, two-dimensional (2D) depth-averaged model is widely used. This is because 2D model allows for significantly reducing the computational time in comparison with three-dimensional models and provides more detailed information of water depth and flow velocities than those obtained in one-dimensional models. Moreover, the water depth is often smaller than the horizontal scales such as the length and width of the channel by a factor of many orders of magnitude, and thus 2D models are widely applied to simulate flow in practical applications (Pham Van et al., 2014a).

Because of low central processor unit (CPU) requirements and simplicity of use, 2D-quasi models can be useful predictive tools, especially in consulting for rivers and stream ecological applications where full 2D models may not be

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[1] *Faculty of Hydrology and Water Resources, Thuyloi University.*

needed and are computationally expensive (Wark et al., 1990, Papanicolaou et al., 2008). The main objective of the present study is to propose a 2D-quasi model that can be used to simulate the flow (e.g. lateral distribution of flow velocity, unit water discharge, and water depth) in compound open-channels. Besides this, the study also aims at (i) accurately representing the experimental data of flow by using the proposed 2D-quasi model and (ii) quantitatively investigating effects of modeling parameters on model-predicted results. The computed results of flow velocity, water depth, unit water discharge, and total water discharge are compared to the observations of four experimental data sets obtained from the literature.

2. METHOD

2.1. Experimental data

Four experimental data sets are used together with a proposed 2D-quasi model for studying the flow in the present consideration. The first experimental data set was reported for a single straight trapezoidal channel (Maynord, 1992) while the last three data sets were performed in compound straight trapezoidal channel (Fraselle, 2010; Zeng et al., 2012). Channel geometry and hydraulic characteristics in all experimental data sets are shown in Fig. 1 and summarized in **Table 1**. In each data set, the water depth was measured and adjusted until the longitudinal water profile in the channel was parallel to the channel bed. Indeed, the flow velocity was taken at different points along vertical axis in the whole or over one-half channel cross-section, and these point velocity measurements were then used to compute the depth-averaged velocity. Detailed information of the experimental data and measurement processes can be found in the relative references mentioned above.

Table 1. Channel geometry and hydraulic characteristics in four experimental data sets

No.	Q (m^3/s)	*H* (cm)	h_{fp} (cm)	B (m)	b_{fp} (m)	s	S_o	R (m)	U_o (m/s)
1	2.86	65.5	0	6.24	0	0.5	0.002	0.493	0.89
2	0.015	6.6	5.08	0.605	0.405	1	0.0019	0.028	0.42
3	0.020	7.27	5.08	0.605	0.405	1	0.0019	0.034	0.45
4	0.0172	9.72	6.54	0.609	0.4456	1	0.00123	0.042	0.31

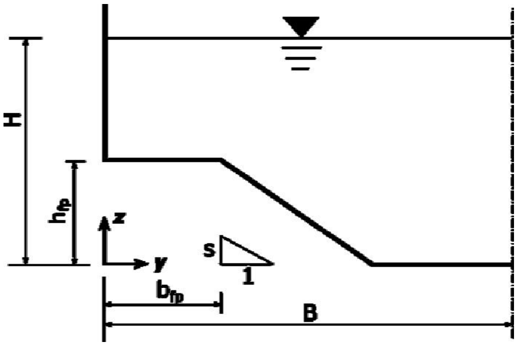

Fig. 1. Geometry of one-half channel cross-section

2.2 Proposed 2D-quasi model

In terms of the proposed 2D-quasi model, the water depth and depth-averaged velocity across a channel section are determined by solving the following equation:

$$\rho g H S_x - B_g \rho \frac{g n^2}{H^{1/3}} U^2 + \frac{\partial}{\partial y}\left(\rho H \nu \frac{\partial U}{\partial y}\right) = 0 \quad (1)$$

where ρ is the water density (= 1000 kg/m^3), g is the gravitational acceleration, H is the water depth (m), S_x is the bed slope in the streamwise direction, $B_g = \sqrt{1 + S_x^2 + S_y^2}$ is the geometrical factor, in which S_y is the bed slope in the lateral direction, n is the Manning coefficient representative of the bed friction, U is the depth-averaged streamwise velocity (m/s), y denotes the lateral direction, and ν is the eddy viscosity (m^2/s).

The eddy viscosity is calculated using an expression based on a non-dimensional eddy viscosity coefficient λ, the shear velocity U_*, and the water depth, under the form

$$\nu = \lambda U_* H, \quad (2)$$

which is known as the zero-equation turbulent model for eddy viscosity.

Eq. (1) is known as 2D-quasi model, in which the gravity, bed shear stress, and turbulent diffusion force are taken into account in order to allow for accurate predictions of lateral distribution of flow velocity and/or of unit water discharge $q = U \times H$.

In terms on numerical implementation, a channel section is divided firstly into a number of nodes (see Fig 2). The latter is determined from simulations in order to obtain the consistent of wetted area of the section at a given water depth. Then, Eq. (1) is solved by using a finite difference method, resulting in a discrete equation system. The latter is solved by using Newton-Raphson iteration method. The free slip condition is applied at channel banks because influences of channel banks are limited to the region close to channel walls (Pham Van et al., 2014b).

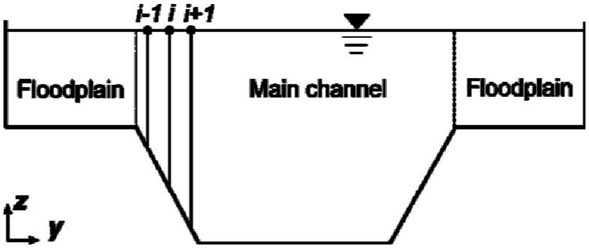

Fig. 2. Finite difference grid nodes using in the model

3. SENSITIVITY ANALYSIS

The third experimental data set (Table 1) is used to investigate sensitivities of modeling parameters. Different constant Manning coefficient and non-dimensional eddy viscosity coefficient are tested to obtain the best fit with the experimental data. The value of each parameter is varied separately whilst keeping the other one constant. Four error estimates including root mean square error (RMSE), mean absolute error (MAE), Nash-Sutcliffe efficient (NSE), and correlation coefficient (r) are applied to access the quality of simulations.

3.1. Manning coefficient

Six simulations were performed by using the same or different values of Manning coefficient in the main channel (n_{mc}) and floodplains (n_{fp}). The value $\lambda = 0.16$ is kept constant in all six simulations. Fig. 3 shows the computed flow velocity from these simulations against the observations while detailed values of error estimates are summarized in Table 2. As shown in Fig. 3 and Table 2, the water depth and flow velocity vary significantly when increasing the value of Manning coefficient, revealing a consistent with results carried out by Pham Van Pham Van et al., 2014b. The model reproduces the reasonable water depth and flow velocity in the channel when using the values $n_{mc} = 0.01$ and $n_{fp} = 0.014$ (corresponding to simulation No. 5). The latters are considered as the best fit values of Manning coefficient among different tested once. The RMSE and MAE of flow velocity are 0.054 and 0.047 m/s, respectively while the NSE and correlation coefficient r are 0.88 and 0.95, respectively.

The best fit values of Manning coefficient are in the expected range of 0.01 for the actual channel bed material. These values are the same as values reported by Fraselle (2010), who used the 2D-Telemac model to reproduce the depth-averaged flow velocity in the same experiment. Indeed, they are also similarly to values carried out by Pham Van et al. (2014b) who applied the 2D-SLIM to investigate the variability of eddy viscosity in open-channels.

An overestimation of flow velocity is observed in the main channel while an underestimation is obtained in the floodplain, especially in the region from the wall to the middle location of the floodplain (Fig. 3). A nearly uniform velocity distribution is also observed in the floodplain. The reason for these may be due to the use of a constant value $\lambda = 0.16$ in calculations, which will be discussed in detail later.

Table 2. Estimate errors when using different values of Manning coefficient

No.	Manning coefficient, *n*		Flow depth, *H* (cm)		Longitudinal velocity, *U* (m/s)			
	Main channel	Floodplain	sim.	obs.	RMSE	MAE	NSE	*r*
1	0.006	0.006	6.04	7.27	0.161	0.125	-0.077	0.918
2	0.010	0.010	7.06		0.060	0.052	0.848	0.934
3	0.016	0.016	8.3		0.114	0.099	0.455	0.945
4	0.01	0.006	6.67		0.135	0.096	0.236	0.835
5		0.014	7.34		0.054	0.047	0.877	0.947
6		0.025	7.85		0.094	0.083	0.629	0.953

3.2. Non-dimensional eddy viscosity coefficient

Different values of non-dimensional eddy viscosity coefficient varying in a range from 0.067 to 2.85 are tested to improve the model-predicted flow velocity. Simulations are started by using a constant value of λ in the both main channel and floodplain, before different values are applied in the either main channel or floodplain. The values n_{mc} = 0.01 and n_{fp} = 0.014 are also kept constant in simulations. Comparisons between computed and observed flow velocity are shown in Fig. 4 while error estimates are summarized in **Table 3**. The computed water depth increases slightly while the flow velocity varies considerably when the value of λ rises from 0.067 to 2.85. The best fit with the observations of flow velocity is obtained when the value $\lambda = 0.16$ and $\lambda = 0.80$ are used in the main channel and floodplain, respectively. The RMSE and MAE of flow velocity from the simulation using these values of λ are 0.034 and 0.027 m/s, respectively while the NSE and correlation coefficient *r* are greater than 0.95.

In comparison with the simulated results obtained when using a constant value $\lambda = 0.16$ in the whole channel cross-section, an improvement of flow velocity is achieved when using $\lambda = 0.16$ in the main channel and $\lambda = 0.80$ in the floodplains (Fig. 4), with the RMSE and MAE decreases twice. In other words, the model-predicted flow velocity is improved considerably when using approximate values of eddy viscosity. This result is consistent with results carried out from the more complex models such as Smagorinsky turbulence closure and k-ε models, which were reported by Pham Van et al. (2014b).

Fig. 4c shows velocity profiles from the 2D-quasi model, 2D-Telemac, and 2D-SLIM against the observations. The velocity profiles from the 2D-Telemac and 2D-SLIM models were reported by Fraselle (2010) and Pham Van et al. (2014b), respectively. Zero-equation turbulence model (with a constant λ = 0.21 in the whole channel) was used to compute eddy viscosity in the 2D-Telemac while Smagorinsky turbulence closure was applied to parameterize eddy viscosity in the 2D-SLIM. It can be observed from Fig. 4c that the velocity profile from the 2D-quasi model is more or less similar those obtained from 2D-Telemac and 2D-SLIM. There is only slight difference of velocity in the main channel and in the region close to channel walls. This discrepancy can be explained by the use of different turbulence closures for eddy viscosity and by the use of different computational models. The difference of velocity in the region close to channel walls is due to the use of boundary condition.

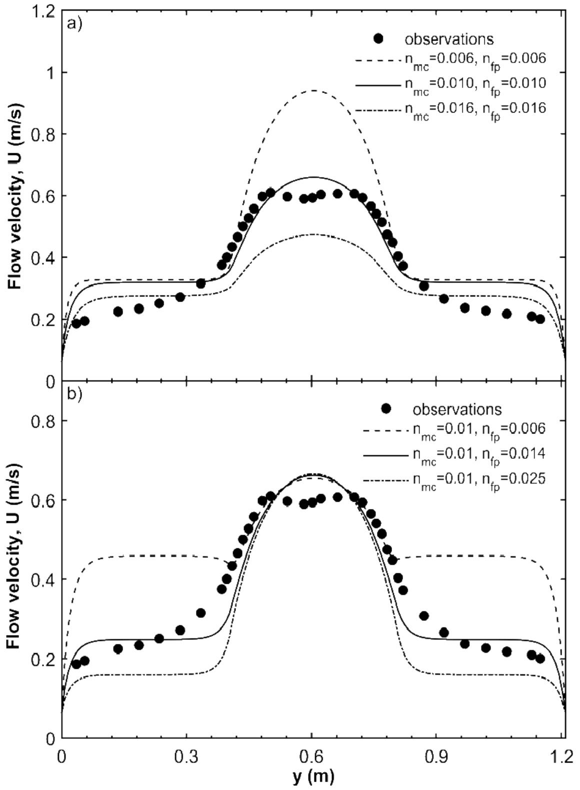

Fig. 3. Lateral distributions of flow velocity: a) using different constant values of n in the channel and b) using different values of n in the floodplains

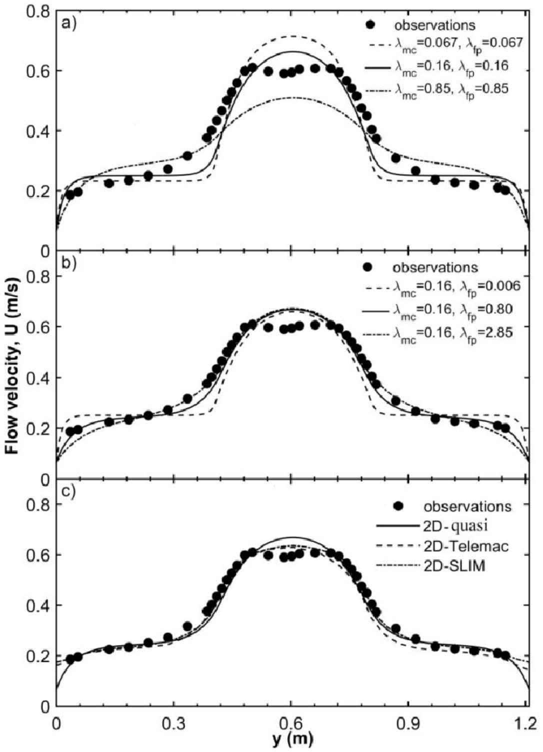

Fig. 4. Lateral distributions of flow velocity when: a) using different constant values of λ in the channel, b) using different values of λ in the floodplains, and c) using different models

Table 3. Estimate errors when using different non-dimensional eddy viscosity coefficient

No.	λ		Flow depth, H (cm)		Longitudinal velocity, U			
	Main channel	Floodplain	sim.	obs.	RMSE	MAE	NSE	*r*
1	0.067	0.067	7.12	7.27	0.072	0.056	0.785	0.932
2	0.16	0.16	7.34		0.054	0.047	0.877	0.947
3	0.85	0.85	7.93		0.077	0.065	0.751	0.977
4	0.16	0.06	7.37		0.065	0.055	0.825	0.925
5		0.80	7.27		0.034	0.027	0.952	0.980
6		2.85	7.21		0.031	0.021	0.961	0.988

4. APPLICATIONS

4.1. Using the first experimental data set

Fig. 5 shows the lateral distribution of simulated and measured flow velocity and unit water discharge in the half channel cross-section of the first experiment. The value $n = 0.033$ is chosen as in the previous study (Maynord, 1992) while $\lambda = 0.16$ is applied (Section 3). It is clearly observed that the model reproduces very well the observations of both flow velocity and unit water discharge. The RMSE and MAE of flow velocity are 0.05 and 0.04 m/s, respectively. These errors are less than 6% of cross-section averaged velocity. The NSE is 0.90 while the correlation *r* between computed and observed flow velocity is 0.95. The RMSE and MAE of unit water discharge are 0.027 and 0.021 m^2/s, respectively. The NSE of unit water discharge is 0.98 while the correlation coefficient *r* is close to unity. The computed water discharge is 2.88 m^3/s which is very close to the measurement value of 2.86 m^3/s.

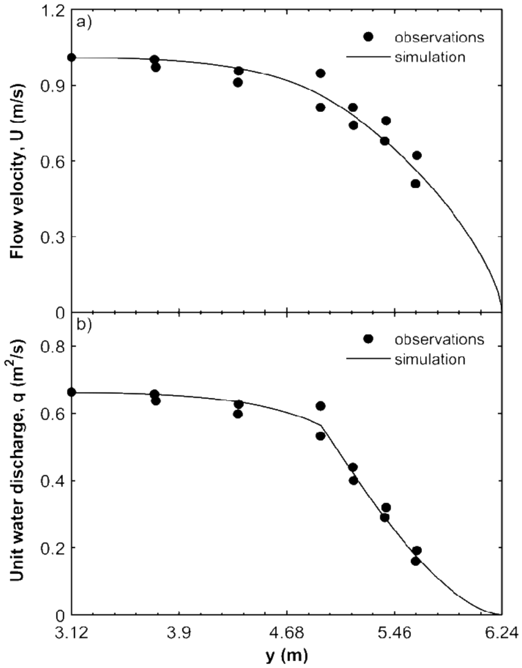

Fig. 5. Simulated and measured: a) flow velocity and b) unit water discharge in a half cross-section channel of the first experimental data set

4.2. Using the second experimental data set

To validate the applicability of the present model to different geometry of channel cross-section, the second experimental data set is performed additionally. The optimal values of n and λ in Section 3 are applied because the second and third experimental data sets were performed exactly in the same channel flume (at the Hydraulics Laboratory of the Université Catholique de Louvain), with the same measurement processes and technique (Fraselle, 2010). This means that the bottom friction for the main channel and floodplain are set equal to 0.01 and 0.014, respectively while the non-dimensional eddy viscosity coefficient equals 0.16 and 0.80 for the main channel and floodplain, respectively.

Similarly to the first experimental data set, the model also reproduces well the lateral distribution of flow velocity and unit water discharge in the second experimental data set in general (Fig. 6). The RMSE and MAE of flow velocity are 0.042 and 0.035 m/s, respectively while these errors of unit water discharge are 0.0019 and 0.0014, respectively. The NSE is 0.94 and 0.98 for the flow velocity and unit water discharge, respectively while the correlation r between computed results and observed data is close to unity for both flow velocity and unit water discharge. The simulated water discharge is 0.0155 m^3/s in comparing to the experimental value of 0.015 m^3/s.

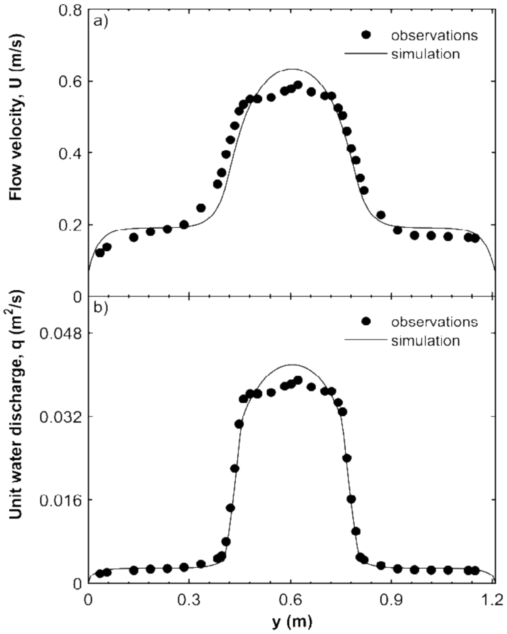

Fig. 6. Lateral distributions of: a) flow velocity and b) unit water discharge in the whole cross-section channel of the second experimental data set

An overestimation of flow velocity is observed in the main channel and floodplains while an underestimation of flow velocity is obtained in the transition region between the main channel and floodplains (Fig. 6a). These discrepancies may be due to the use of a simple model such as the zero-equation turbulence model for eddy viscosity in calculations. As noticed in the previous studies (Pham Van et al., 2014b; Zeng et al., 2012), eddy viscosity can vary significantly in the whole channel section. Small values of eddy viscosity can occur around in the middle location of floodplain and central channel while large values can appear around the transition locations between the main channel and floodplain. However, these characteristics of eddy viscosity cannot be captured by using the zero-equation turbulence model. Thus, the use of zero-equation turbulence model for eddy viscosity can be a reason for the discrepancies in flow velocity and consequently unit water discharge.

4.3. Using the fourth experimental data set

An additional simulation has been performed by using the remaining experiment reported in Section 2, to further demonstrate the applicability of the proposed model. The bottom friction n_{mc} = 0.009 and n_{fp} = 0.02 are applied in the main channel and floodplains, respectively (Pham Van et al., 2014b; Zeng et al., 2012). The approximate values of λ obtained in Section 2 are also used. Comparisons between computed and observed flow velocity and unit water discharge in a half channel cross-section are given in Fig. 7, revealing that a very good agreement is obtained. The RMSE and MAE of flow velocity are 0.019 and 0.014 m/s, respectively. The NSE and correlation coefficient *r* between simulated and observed flow velocity are greater than 0.97. The RMSE and MAE of unit water discharge equal 0.0013 and 0.0007 m^2/s, respectively. The NSE and correlation coefficient *r* are close to unity. Similarly to simulation of the second experiment, the model predicts an overestimation of flow velocity and consequently unit water discharge in the main channel.

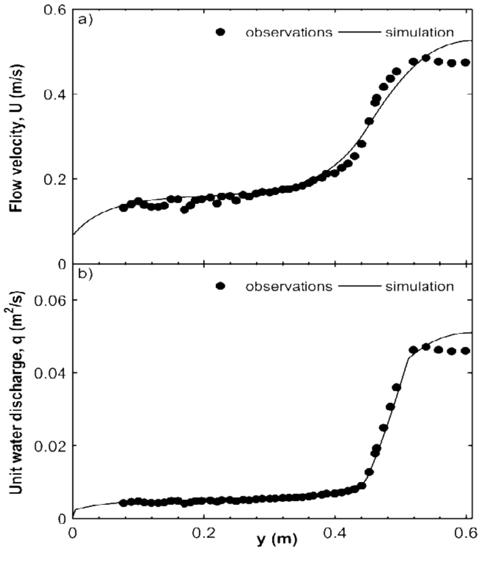

Fig. 7. Lateral distributions of: a) flow velocity and b) unit water discharge in the half cross-section channel of the fourth experimental data set

5. CONCLUSION

Flows in compound open-channels often exhibit complex characteristics in terms of lateral distribution of flow velocity and water depth due to the variation of section's geometry and topography, bed friction, and transversal transfer of momentum between floodplains and main channel. The aims of the present study were to (i) propose a 2D-quasi model that is capable of predicting lateral distribution of flow velocity and/or of unit water discharge, (ii) explore the effects of modeling parameters on model-predicted results, and (iii) reproduce the experimental data of flow by using the proposed 2D-quasi model.

The results clearly showed that, firstly, the proposed 2D-quasi model was successfully applied for all four experimental data sets of the flow in trapezoidal open-channels. The RMSE and MAE of flow velocity were less than 10% of cross-section averaged velocity while Nash-Sutcliffe efficient and correlation coefficient were close to unity. Moreover, discrepancies between simulated results and experiments were vicinity for both water depth and water discharge. Secondly, both Manning coefficient and non-dimensional eddy viscosity coefficient affected significantly on the model-predicted results. Finally, there was no significant difference of model-predicted velocity from the proposed 2D-quasi model and fully 2D-Telemac and 2D-SLIM for the particular applications of the compound straight trapezoidal channel presented in this study.

The proposed 2D-quasi model in the present study is believed to be a useful tool for (i) building stage-discharge relationship in both small-scale and large-scale applications and (ii) calculating bedload sediment transport through cross-sections in the next step of the research.

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Tóm tắt:

MÔ HÌNH BÁN 2 CHIỀU CHO MÔ PHỎNG DÒNG CHẢY TRONG KÊNH HỞ

Bài báo này đề xuất mô hình bán 2 chiều dùng cho tính toán mô phỏng phân bố vận tốc và lưu lượng dòng chảy đơn vị trong các mặt cắt của kênh hở. Phân bố vận tốc và lưu lượng dòng chảy đơn vị được xác định bằng cách giải phương trình đặc trưng mà nó (i) được biến đổi từ các phương trình Reynolds và (ii) cho phép xem xét cả trọng lực, lực ma sát đáy và lực khuếch tán do dòng chảy rối trong tính toán. Trước tiên, phân tích độ nhạy về các thông số của mô hình (bao gồm hệ số nhám Mannining và hệ số nhớt) đã được thực hiện bằng cách sử dụng một bộ số liệu đo đạc dòng chảy thực nghiệm. Sau đó, các thông số của mô hình đã được hiệu chỉnh trước khi mô hình được kiểm định. Tiếp theo, mô hình đề xuất bán 2 chiều được áp dụng để tái hiện lại dòng chảy đo đạc của ba bộ số liệu thực nghiệm khác. Bốn tiêu chí sai số khác nhau đã được tính toán để đánh giá định lượng chất lượng của kết quả tính toán, thể hiện (i) sự phù hợp tốt giữa kết quả tính toán và đo đạc và (ii) mô hình đề xuất bán 2 chiều đã áp dụng thành công trong việc tái hiện lại dòng chảy đo đạc của cả bốn bộ số liệu thực nghiệm. Cuối cùng, khả năng của mô hình đề xuất bán 2 chiều cũng được thảo luận.

Từ khoá: Kênh hở hỗn hợp, mô hình bán 2 chiều, vận tốc dòng chảy, độ nhớt.

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