ANALYSIS OF FREE AND FORCED VIBRATIONS IN THE TWO-STOREY BUILDING FRAME WITH CRACKS

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ABSTRACT

This paper aims to analyze the free and forced vibrations of the two-storey building frame with cracks. The two-storey frame system with cracks is modelled when many constituent parameters existed such as crack depth, crack location, flexural stiffness and elastic force which were involved by a nonlinear function. In addition, in analysis of forced vibrations, the effect of random external loads is carried out to find probability characteristics of displacements. The resulting of structural responses is the mean square value of the displacement, the natural frequency of vibrations, and the probability density function of displacements. It can be found that when cracks appear in the middle of the columns, they would need to be considered most carefully because this problem will make the structure to become very dangerous during its existence. The results obtained in this paper can be used in the test calculation taking into account the influence of many parameters when analyzing responses of the structural system.

PHÂN TÍCH DAO ĐỘNG TỰ DO VÀ CƯỠNG BỨC TRONG KHUNG NHÀ HAI TẦNG CÓ VẾT NỨT

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TỪ KHÓA

Khung hai tầng
Vết nứt
Phân tích phi tuyến
Tần số tự nhiên
Chuyển vị ngang

TÓM TẮT

Bài báo này phân tích dao động tự do và dao động cưỡng bức của khung nhà 2 tầng có vết nứt. Hệ khung hai tầng có vết nứt được mô hình hóa khi tồn tại nhiều tham số cấu thành như độ sâu vết nứt, vị trí vết nứt, độ cứng uốn và lực đàn hồi tham gia bởi một hàm phi tuyến. Ngoài ra, trong phân tích dao động cưỡng bức, tác dụng của tải trọng ngoài ngẫu nhiên được thực hiện để tìm các đặc trưng xác suất của chuyển vị. Kết quả của phản ứng kết cấu là giá trị bình phương trung bình của chuyển vị, tần số dao động tự nhiên và hàm mật độ xác suất của chuyển vị. Có thể thấy rằng khi các vết nứt xuất hiện ở giữa các cột thì cần phải xem xét kỹ lưỡng nhất bởi vấn đề này sẽ làm cho kết cấu trở nên rất nguy hiểm trong quá trình tồn tại. Các kết quả thu được trong bài báo này có thể được sử dụng trong tính toán thử nghiệm có xét đến ảnh hưởng của nhiều thông số khi phân tích phản ứng của hệ kết cấu.

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1. Introduction

Structural modeling with cracks has been studied by many authors theoretically and experimentally for decades. Theoretically, the authors in [1] - [3] propose a spring model to describe the crack. The authors [4], [5] have developed a spring crack model from the experimental formula results. The above research results play a very important role to form the theory of structural behavior with cracks in the form of springs with equivalent stiffness considering the depth and location of the cracks. In the document [6], the author mentioned the influence of cracks on the natural frequency in beams. Theoretically, when calculating, usually describe the crack through linear vibrations [6] - [8]. In [9], the influence of nonlinear stiffness on the stiffness of the structure with cracks was studied. The above results are fundamental to develop structural models with cracks in the form of linear or nonlinear vibration problems.

The analysis of the behavior of the structure in the nonlinear problem under the influence of random loads has been mentioned many times in the studies [10] - [13]. The main tools for analyzing the behavior of a nonlinear problem under the influence of random excitation lead to an equivalent linearization problem to find the probabilistic features of the displacement. In the classical studies on equivalent linearization [12], [13], the authors have obtained many reliable results to compare with those calculated by exact calculation method [14], [15]. Recently, in the literature [10], [16], the equivalent linearization method is mentioned, the results obtained are quite satisfactory.

Modeling of structures with cracks under the influence of harmonic forces describes the linear and nonlinear problems studied in [9]. However, in [9] there is only a very primitive problem about the behavior of the structure when changing the flexural stiffness. The combination of structural modeling with cracks under the influence of random excitations has not been studied. This paper proposes a structural model with cracks subjected to random excitation in the form of a nonlinear problem of a two-degree-of-freedom system such as a two-storey building frame structure. As a result, the behavior of the structural system has been determined in two cases: (1) Free vibrations – finding the natural frequency and (2) Forced vibrations – finding the probabilistic characteristic of displacement, thereby determining the influence of crack factors such as crack location, crack depth, crack stiffness on the displacement of the structure. The paper used some classical equivalent linearization methods found in [11], [12] as well as a new method [10], [16] to solve the nonlinear problems. The calculated results herein have been compared with the exact solution determined by the well-known Fokker-Planck equation [14], [15].

The contents of this paper consist of sections: 1. Introduction; 2. The method of vibration analysis; 3. Results and discussion; 4. Conclusion.

2. The method of vibration analysis

2.1. Modelling the two-storey building frame with cracks

A two-storey building frame is modeled, as shown in Figure 1a. An each floor has mass $m_1 = m$, $m_2 = m$, respectively. The frame has total height $2H$. The floors are considered to have infinite stiffness, and two columns have four bending stiffness at the pieces are EI_1, EI_2, EI_3 *and* EI_4, respectively. The damping force has a viscous coefficient of $c_1 = c$, $c_2 = c$ corresponding to the each floor. We assume that the frame is to be subjected to the environmental random loads P_{f1}, P_{f2} at the level of each floor.

For a story of height H and a column with modulus E and second moment of area I_c, the lateral stiffness of a column with clamped ends, implied by the shear-building idealization, is $12EI_c/h^3$. So that the right column has lateral stiffness as:

$$k_2 = \frac{12EI_2}{H^3}, k_4 = \frac{12EI_4}{H^3}, \tag{1}$$

For the left column with cracks, the lateral stiffness will be calculated by the method of initial parameters, that detail results have taken in [7], we will get the result as

$$k_1 = \frac{12EI_1}{H^3}.\frac{(1+\beta_1)}{\left(4-12\alpha_1+12\alpha_1^2+\beta\right)}; k_3 = \frac{12EI_3}{H^3}.\frac{(1+\beta_2)}{\left(4-12\alpha_2+12\alpha_2^2+\beta_2\right)}; \quad \beta_1 = \frac{k_{cr1}H}{EI_1}; \beta_2 = \frac{k_{cr2}H}{EI_3} \quad (2)$$

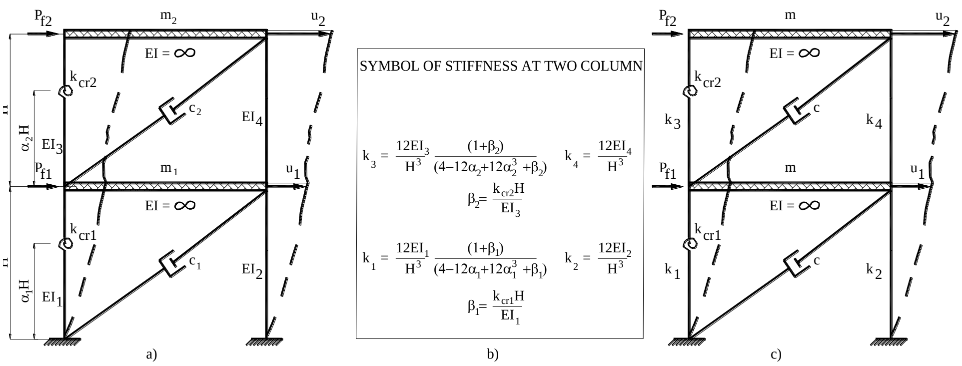


Figure 1. *Model of two-storey building frame with cracks in the column*

The cracks (at the position $\alpha_i H$ ($0\leq\alpha\leq1$), $i=1\div2$) is modeled as an elastic spring with the equivalent stiffness k_{cr1} and k_{cr2}. If the column has cross-sectional area to be rectangular b×h and a is the depth of the crack, then $k_{cr(i)}$ is shown in following [5].

$$k_{cr} = \frac{bh^2E}{36\pi\mu^2\left(0.5033-0.9022\mu+3.412\mu^2-3.18\mu^3+5.793\mu^4\right)}; \quad \mu = \frac{2a}{h} \quad (3)$$

In this paper, we assume that at the floor level the elastic (or inelastic) resisting force contains with two nonlinear values of $g_1(u_1, u_2)$ and $g_2(u_1, u_2)$ to obtain the equations of motion in the system of two-degrees of freedom as

$$m\ddot{u}_1 + c\dot{u}_1 + r_{11}u_1 + g_1(u_1, u_2) = P_{f1}; \; m\ddot{u}_2 + c\dot{u}_2 + r_{22}u_2 + g_2(u_1, u_2) = P_{f2} \quad (4)$$

where $r_{11} = k_1 + k_2 + k_3 + k_4;\ r_{22} = k_3 + k_4;$ Here we assume that the nonlinear parts $g_1(u_1, u_2)$ and $g_2(u_1, u_2)$ are given in the following $g_1(u_1,u_2)/m = 4\gamma_1 u_1^3 + 2\gamma_3 u_1 u_2^2, g_2(u_1,u_2)/m = 4\gamma_5 u_2^3$ with $\gamma_1, \gamma_3, \gamma_5$ are positive constants. By some manipulations the equations of motion in the system (4) became

$$\ddot{u}_1 + h\dot{u}_1 + \chi_{11}u_1 + 4\gamma_1 u_1^3 + 2\gamma_3 u_1 u_2^2 = \varpi_1(t); \quad \ddot{u}_2 + h\dot{u}_2 + \chi_{22}u_2 + 4\gamma_5 u_2^3 = \varpi_2(t) \quad (5)$$

where $h = c/m; \chi_{ii} = r_{ii}/\text{m}; P_{f1}/m = \varpi_1(t); P_{f2}/m = \varpi_2(t);$ Here $\varpi_1(t), \varpi_2(t)$ are zero-mean Gaussian white noise stationary random processes with the spectral density taken a constant value of S_0.

2.2. Exact solution

In equations (5), $U(u_1,u_2)$ is the potential energy expressed by

$$U(u_1,u_2) = \frac{1}{2}\chi_{11}u_1^2 + \frac{1}{2}\chi_{22}u_2^2 + \gamma_1 u_1^4 + \gamma_3 u_1^2 u_2^2 + \gamma_5 u_2^4 \quad (6)$$

Equations (5) can be rewritten by

$$\ddot{u}_i + h\dot{u}_i + \frac{\partial}{\partial u_i}U(u_1,u_2) = \varpi_i(t), \qquad i = 1,2 \quad (7)$$

The exact solution of equations (7) can be made by the Fokker-Planck equations [14], [15] and then given an exact stationary probability density function (PDF) $p(u_1,u_2)$ as following

$$p(u_1,u_2)=C\exp\left\{-\frac{h}{\pi S_0}U(u_1,u_2)\right\},\qquad C=\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\exp\left\{-\frac{h}{\pi S_0}U(u_1,u_2)\right\}du_1du_2\right)^{-1} \tag{8}$$

We will get the exact mean square responses by

$$E\left\{u_i^2\right\}_{ex}=C\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\mathrm{u}_i^2\exp\left\{-\frac{h}{\pi S_0}U(u_1,u_2)\right\}du_1du_2,\qquad i=1,2 \tag{9}$$

2.3. Approximate solution by equivalent linearization methods

2.3.1. Forced vibrations

There are many ways to approximate the nonlinear equations (5) by means of equivalent linearization. Here we use the conventional linearization method combined with LOMSEC [10], [16]. By utilizing equations, (5) can be rewritten in matrix form as

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\ddot{u}_1\\\ddot{u}_2\end{bmatrix}+\begin{bmatrix}h&0\\0&h\end{bmatrix}\begin{bmatrix}\dot{u}_1\\\dot{u}_2\end{bmatrix}+\begin{bmatrix}\chi_{11}&0\\0&\chi_{22}\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}+\begin{bmatrix}4\gamma_1u_1^3+2\gamma_3u_1u_2^2\\4\gamma_5u_2^3\end{bmatrix}=\begin{bmatrix}\varpi_1(t)\\\varpi_2(t)\end{bmatrix} \tag{10}$$

$$M\ddot{u}+C\dot{u}+Ku+\Phi=\varpi(t) \tag{11}$$

where

$$M=\begin{bmatrix}1&0\\0&1\end{bmatrix},C=\begin{bmatrix}h&0\\0&h\end{bmatrix},\ K=\begin{bmatrix}\chi_{11}&0\\0&\chi_{22}\end{bmatrix},\ \Phi=\begin{bmatrix}\Phi_1\\\Phi_2\end{bmatrix}=\begin{bmatrix}4\gamma_1u_1^3+2\gamma_3u_1u_2^2\\4\gamma_5u_2^3\end{bmatrix},\ u=\begin{bmatrix}u_1\\u_2\end{bmatrix}.\ \dot{u}=\begin{bmatrix}\dot{u}_1\\\dot{u}_2\end{bmatrix}.\ddot{u}=\begin{bmatrix}\ddot{u}_1\\\ddot{u}_2\end{bmatrix} \tag{12}$$

The equivalent linearization system is obtained in the form

$$\begin{bmatrix}1&0\\0&1\end{bmatrix}\begin{bmatrix}\ddot{u}_1\\\ddot{u}_2\end{bmatrix}+\begin{bmatrix}h&0\\0&h\end{bmatrix}\begin{bmatrix}\dot{u}_1\\\dot{u}_2\end{bmatrix}+\begin{bmatrix}\chi_{11}+k_{11}^e&k_{12}^e\\k_{21}^e&\chi_{22}+k_{22}^e\end{bmatrix}\begin{bmatrix}u_1\\u_2\end{bmatrix}=\begin{bmatrix}\varpi_1(t)\\\varpi_2(t)\end{bmatrix} \tag{13}$$

The difference ε between the original nonlinear equations (10) or (11) with the linear system (13) is

$$\varepsilon=\Phi-K^eu \tag{14}$$

where
$$\varepsilon=\begin{bmatrix}\varepsilon_1\\\varepsilon2\end{bmatrix},\ K^e=\begin{bmatrix}k_{11}^e&k_{12}^e\\k_{21}^e&k_{22}^e\end{bmatrix},\ \Phi=\begin{bmatrix}\Phi_1\\\Phi_2\end{bmatrix}=\begin{bmatrix}4\gamma_1u_1^3+2\gamma_3u_1u_2^2\\4\gamma_5u_2^3\end{bmatrix},\ u=\begin{bmatrix}u_1\\u_2\end{bmatrix} \tag{15}$$

The components of K^e can be taken as following

$$\begin{aligned}
&\frac{\partial}{\partial k_{11}^e}E\left[\varepsilon_1^2\right]=-4\gamma_1E\left[u_1^4\right]-2\gamma_3E\left[u_1^2u_2^2\right]+k_{11}^eE\left[u_1^2\right]+k_{12}^eE\left[u_1u_2\right]=0\\
&\frac{\partial}{\partial k_{12}^e}E\left[\varepsilon_1^2\right]=-4\gamma_1E\left[u_1^3u_2\right]-2\gamma_3E\left[u_1u_2^3\right]+k_{11}^eE\left[u_1u_2\right]+k_{12}^eE\left[u_1^2\right]=0\\
&\frac{\partial}{\partial k_{21}^e}E\left[\varepsilon_2^2\right]=-4\gamma_5E\left[u_1u_2^3\right]+k_{21}^eE\left[u_1^2\right]+k_{22}^eE\left[u_1u_2\right]=0\\
&\frac{\partial}{\partial k_{22}^e}E\left[\varepsilon_2^2\right]=-4\gamma_5E\left[u_2^4\right]+k_{21}^eE\left[u_1u_2\right]+k_{22}^eE\left[u_2^2\right]=0
\end{aligned} \tag{16}$$

Assuming that u_1 and u_2 are independent and then $E[u_1^{2n+1}u_2^{2m+1}]=0$, and following [16] we can give $k_{12}^e(r)=k_{21}^e(r)=0$ and

$$k_{11}^e(r) = \frac{4\gamma_1 E\left[u_1^4\right] + 2\gamma_3 E\left[u_1^2 u_2^2\right]}{E\left[u_1^2\right]} = 4\gamma_1 E\left\{u_1^2\right\}\frac{T_{2,r}}{T_{1,r}} + 2\gamma_3 E\left\{u_2^2\right\}\frac{T_{1,r}}{T_{0,r}};\ k_{22}^e(r) = \frac{4\gamma_5 E\left[u_2^4\right]}{E\left[u_1^2\right]} = 4\gamma_5 E\left\{u_2^2\right\}\frac{T_{2,r}}{T_{1,r}} \quad (17)$$

$$T_{0,r} = \int_0^r \eta(t)dt,\ \ T_{1,r} = \int_0^r t^2\eta(t)dt,\ \ T_{2,r} = \int_0^r t^4\eta(t)dt,\ \ \eta(t) = \frac{1}{\sqrt{2\pi}}e^{-t^2/2} \quad (18)$$

By utilizing with r→∞, equations (17) and (18) will get k_{11}^e, k_2^e by the conventional linearization method as following

$$k_{11}^e = 12\gamma_1 E\left\{u_1^2\right\} + 2\gamma_3 E\left\{u_2^2\right\};\quad k_{22}^e = 12\gamma_5 E\left\{u_2^2\right\} \quad (19)$$

After determining the coefficients of K^e we will find approximate mean square responses for equivalent linearization equations (13). By the assumption, $\varpi_1(t), \varpi_2(t)$ are zero-mean Gaussian white noise stationary random processes with the spectral densitytaken a constant value of S_0.

The matrix of frenquency response function to the linear system (13) is

$$\alpha(\omega) = \left[-\omega^2 M + i\omega C + (K + K^e)\right]^{-1} \quad (20)$$

where $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ C = \begin{bmatrix} h & 0 \\ 0 & h \end{bmatrix},\ K = \begin{bmatrix} \chi_{11} & 0 \\ 0 & \chi_{22} \end{bmatrix},\ K^e = \begin{bmatrix} k_{11}^e & 0 \\ 0 & k_{22}^e \end{bmatrix},\ S_\varpi(\omega) = \begin{bmatrix} S_0 & 0 \\ 0 & S_0 \end{bmatrix}$ (21)

$$\alpha(\omega) = \frac{1}{L(\omega)}\begin{bmatrix} -\omega^2 + i\omega h + \chi_{22} + k_{22}^e & 0 \\ 0 & -\omega^2 + i\omega h + \chi_{11} + k_{11}^e \end{bmatrix} \quad (22)$$

$$\begin{gathered} L(\omega) = \Lambda(i\omega) = \left(-\omega^2 + i\omega h + \chi_{11} + k_{11}^e\right)\left(-\omega^2 + i\omega h + \chi_{22} + k_{22}^e\right) = \lambda_4(i\omega)^4 + \lambda_3(i\omega)^3 + \lambda_2(i\omega)^2 + \lambda_1(i\omega) + \lambda_0 \\ \lambda_4 = 1,\ \lambda_3 = 2h,\ \ \lambda_2 = \chi_{11} + k_{11}^e + \chi_{22} + k_{22}^e + h^2,\ \lambda_1 = h(\chi_{11} + k_{11}^e + \chi_{22} + k_{22}^e),\ \lambda_0 = (\chi_{11} + k_{11}^e)(\chi_{22} + k_{22}^e) \end{gathered} \quad (23)$$

And then obtaining a matrix of mean square elements as following

$$E\left\{uu^T\right\} = \int_{-\infty}^{\infty} \alpha(-\omega)S_\varpi(\omega)\alpha^T(\omega)\,\mathrm{d}\omega,\ \ E\left\{u_1^2\right\} = \int_{-\infty}^{\infty}\frac{\Xi_{11}(\omega)}{\Lambda(\omega)\Lambda(-\omega)}\mathrm{d}\omega,\ \ E\left\{u_2^2\right\} = \int_{-\infty}^{\infty}\frac{\Xi_{22}(\omega)}{\Lambda(\omega)\Lambda(-\omega)}\mathrm{d}\omega \quad (24)$$

where

$$\begin{gathered} \Xi_{11}(\omega) = S_0(-\omega^2 - i\omega h + \chi_{22} + k_{22}^e)(-\omega^2 + i\omega h + \chi_{22} + k_{22}^e) = \xi_2\omega^4 + \xi_1\omega^2 + \xi_0 \\ \textit{where}\quad \xi_2 = S_0,\ \ \xi_1 = S_0(h^2 - 2(\chi_{22} + k_{22}^e)),\ \ \xi_0 = S_0((\chi_{22} + k_{22}^e)^2) \\ \Xi_{22}(\omega) = S_0(-\omega^2 - i\omega h + \chi_{11} + k_{11}^e)(-\omega^2 + i\omega h + \chi_{11} + k_{11}^e) = \zeta_2\omega^4 + \zeta_1\omega^2 + \zeta_0 \\ \textit{where}\quad \zeta_2 = S_0,\ \ \zeta_1 = S_0(h^2 - 2(\chi_{11} + k_{11}^e)),\ \ \zeta_0 = S_0((\chi_{11} + k_{11}^e)^2) \end{gathered} \quad (25)$$

Using the Cramer's formula in [13] to equations (24) will get mean square results as following

$$\begin{gathered} E\left\{u_1^2\right\} = \frac{\pi}{\lambda_4}\begin{vmatrix} 0 & \xi_2 & \xi_1 & \xi_0 \\ -\lambda_4 & \lambda_2 & -\lambda_0 & 0 \\ 0 & -\lambda_3 & \lambda_1 & 0 \\ 0 & \lambda_4 & -\lambda_2 & \lambda_0 \end{vmatrix}\begin{vmatrix} \lambda_3 & -\lambda_1 & 0 & 0 \\ -\lambda_4 & \lambda_2 & -\lambda_0 & 0 \\ 0 & -\lambda_3 & \lambda_1 & 0 \\ 0 & \lambda_4 & -\lambda_2 & \lambda_0 \end{vmatrix}^{-1} \\ E\left\{u_2^2\right\} = \frac{\pi}{\lambda_4}\begin{vmatrix} 0 & \zeta_2 & \zeta_1 & \zeta_0 \\ -\lambda_4 & \lambda_2 & -\lambda_0 & 0 \\ 0 & -\lambda_3 & \lambda_1 & 0 \\ 0 & \lambda_4 & -\lambda_2 & \lambda_0 \end{vmatrix}\begin{vmatrix} \lambda_3 & -\lambda_1 & 0 & 0 \\ -\lambda_4 & \lambda_2 & -\lambda_0 & 0 \\ 0 & -\lambda_3 & \lambda_1 & 0 \\ 0 & \lambda_4 & -\lambda_2 & \lambda_0 \end{vmatrix}^{-1} \end{gathered} \quad (26)$$

From equation (26) and equation (19), we will find out as unknowns $E\left\{u_1^2\right\}, E\left\{u_2^2\right\}$ and called these results as $E\left\{u_1^2\right\}_{\text{co}}, E\left\{u_2^2\right\}_{\text{co}}$ to compare with the exact results $E\left\{u_1^2\right\}_{ex}, E\left\{u_2^2\right\}_{ex}$ determined by the equation (9).

2.3.2. Free vibrations

After obtaining the results $E\left\{u_1^2\right\}, E\left\{u_2^2\right\}$ in the above section, we will proceed to determine the natural frequencies of the system by the following frequency equation:

$$\det\left[(K+K^e)-\omega^2 M\right]=0 \tag{27}$$

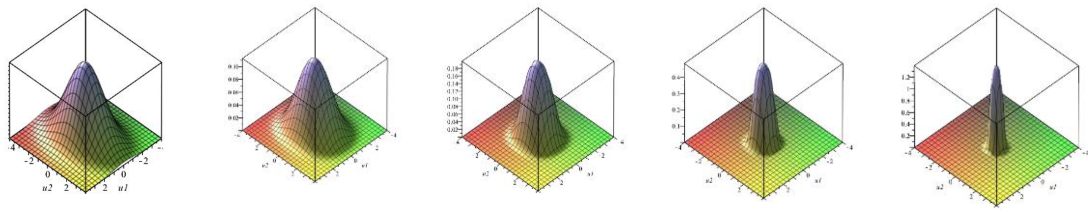
Corresponding to the results $E\left\{u_1^2\right\}_{ex}, E\left\{u_2^2\right\}_{ex}$ we will get two natural friequencies that called $(\omega_1)_{GT}, (\omega_2)_{GT}$ and corresponding to the results $E\left\{u_1^2\right\}_{\text{co}}, E\left\{u_2^2\right\}_{\text{co}}$ we also will get two natural friequencies called $(\omega_1)_{co}, (\omega_2)_{co}$. These frequency results are shown in the tables 1 and 2.

3. Results and discussion

Here we will consider a two-storey reiforced-concrete building frame with assuming that this frame has masless and lumped weight at the each floor level as *m=44.5 kN*. We will take the initial values as follows: $E = 2.10^5\ kN/m^2$, $S_0 = 1$, $h = 0.5$, $bxh = 30x36\ cm^2$, $H = 3.6\ m$, $a_1 = 11\ cm$, $a_2 = 9\ cm$, $\alpha_1 = 0.7$, $\alpha_1 = 0.8$, $\gamma_1 = \gamma_3 = \gamma_5 = 1$. When changing one parameter, the other parameters are taken initially as above. We will calculate for 3 cases of parameters change as follows: (3.1) *When changing the nonlinear coefficient of* γ, (3.2) *When changing the crack position* α_1, α_2, (3.3) *When changing the spectral density of* S_0. In each case, there are results including: Mean square $E\left\{u_1^2\right\}, E\left\{u_2^2\right\}$ of u_1, u_2; Two natural frequencies of ω_1 and ω_2; Shown the figures of PDF $p(u_1,u_2)$;

3.1. When changing the nonlinear coefficient of γ

When changing $\gamma=0.001$, $\gamma=0.01$, $\gamma=1$, $\gamma=10$, $\gamma=100$ and from equations (8), (9) and (26), we can get the results on Figure 2 and Table 1.



$\gamma=0.001$ $\gamma=0.1$ $\gamma=1$ $\gamma=10$ $\gamma=100$

Figure 2. *PDF $p(u_1,u_2)$ for 5 cases when changing γ*

Table 1. *Mean square of u_1 versus $\gamma_1=\gamma_3=\gamma_5=\gamma$*

γ	$E\left\{u_1^2\right\}_{\text{ex}}$	$E\left\{u_1^2\right\}_{\text{co}}$	**Error (%)**	$E\left\{u_2^2\right\}_{\text{ex}}$	$E\left\{u_2^2\right\}_{\text{co}}$	**Error (%)**
0.001	1.24834873	1.24833807	0.001	2.51447849	2.51672479	0.087
0.1	0.984986	0.970525167	1.468	1.50102759	1.48074351	1.468
1	0.542385494	0.508084060	6.325	0.652282036	0.628096693	3.7070
10	0.214773491	0.193022864	10.127	0.229226511	0.218776905	4.5587
100	0.0733504427	0.0647709160	11.696	0.0749315750	0.0713397666	4.7937

Figure 2 shows the probability density function $p(u_1,u_2)$ corresponding to 5 cases of changing γ. These maps in figure 2 have been shown clearly the degree of dispersion of values according to the nonlinear coefficient γ. Table 1 is the mean square value of displacement u_1 and u_2 for 5 cases γ. With these results, when the nonlinear coefficient is small, the error values according to the two methods will be accurate – it can be found that if the system is more nearly linear and then the calculation by two methods is more exactly. The nonlinear coefficient γ is inversely proportional to the mean square values of u_1 and u_2.

3.2. When changing the crack position α_1, α_2

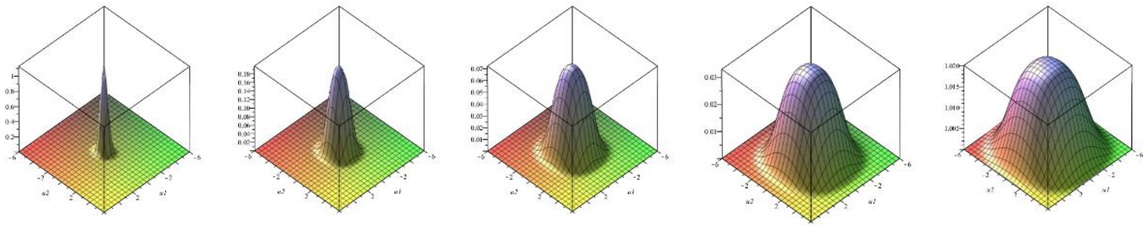
Table 2 shows the two values of natural frequency changes when changing the crack position. Looking at the results in Table 2 shows that the cracks in the middle of the column are very important in the analysis of the structure with cracks. The calculation results by two methods show that the natural frequency value will be less effective at the middle of the column because error is very big. This phenomenon occurs because in the middle of the column there is the greatest bend.

Table 2. *Natural frequencies of ω_1 and ω_2 with $\alpha_1=\alpha_2=\alpha$*

α	$(\omega_1)_{GT}$	$(\omega_1)_{co}$	Error (%)	$(\omega_2)_{GT}$	$(\omega_2)_{co}$	Error (%)
0.1	3.19706307	3.15103305	1.440	3.54752457	3.47655456	2.001
0.3	3.21712847	3.17341453	1.359	3.59007735	3.52696249	1.758
0.45	3.22575655	3.18294923	1.327	3.60951365	3.54959951	1.660
0.5	3.61096259	3.18364384	11.834	3.22638695	3.55127830	10.070
0.6	3.22390963	3.18091264	1.334	3.60529164	3.54470101	1.681
0.8	3.20761156	3.16283605	1.396	3.56945310	3.50268612	1.871
0.9	3.19706307	3.15103305	1.440	3.54752457	3.47655456	2.001

3.3. When changing the spectral density of S_0

Figure 3 shows the function $p(u_1,u_2)$ corresponding to 5 cases of changing S_0=0.1, S_0=1, S_0=5, S_0=20 and S_0=50. As shown in this figure, we can see clearly the degree of dispersion of values $p(u_1,u_2)$ when changing values of S_0.



S_0=0.1 *S_0=1* *S_0=5* *S_0=20* *S_0=50*

Figure 3. *PDF p(u1,u2) for 5 cases when changing S0*

4. Conclusions

The paper has proposed a structural model of a 2-storey building frame with cracks subjected to random excitation of white noise considering the influence of the nonlinear structural system. By modeling the structure as a two-degree-of-freedom system, results were carried out in two problems of forced vibration and free vibration. The calculation results obtained here partly reflect the close working with a real construction. From these results, the following observations can be drawn: (1) The degree of nonlinearity in the response of the system cannot be ignored. The higher the degree of nonlinearity is, the more complex it is. (2) Occurrence of cracks at the mid-column position has an important influence on the behavior of the structure. (3) The influence of spectral density is very important during structural analysis. The limitation in this paper is that we only consider the standard Gaussian process, no correlation of u_1 with u_2 is a limited assumption, which suggests for further research.

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